Heterogeneous Expectations and Macroeconomic Stability under Monetary and Fiscal Policy Rules*

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Abstract

The existing literature on macroeconomic policy rules emphasizes the stabilizing properties of two alternative regimes: one of "monetary dominance"—whereby the central bank fights inflation aggressively and fiscal authorities stabilize public debt, and one of "fiscal dominance"—whereby fiscal authorities disregard debt stabilization and the central bank passively accommodates inflation taxes to rule out debt escalation. A common feature of these studies is the assumption of forward looking agents with rational expectations. The contribution of this paper is to explore monetary-fiscal interactions in a New Keynesian model with boundedly rational agents and heterogeneous expectations. An endogenous evolutionary selection of forecasting strategies is operative, with agents switching toward more accurate rules in line with "learning-to-forecast" experimental evidence. We show that neither the Taylor principle nor active fiscal policy in line with the fiscal theory of the price level guarantees uniqueness of equilibrium. Under both monetary and fiscal dominance, heterogeneous expectations easily open the door to aggregate instability in the form of either multiple attracting cycles or perpetual fluctuations. We also contribute to the quest for stabilizing policy rules. Our analysis indicates that a regime of monetary dominance whereby primary surpluses respond aggressively to debt accumulation is likely to ensure macroeconomic stability and avoid complex dynamics, for the sufficiently strong fiscal reaction has the effect of anchoring expectations and coordinating the evolutionary dynamics of adaptive beliefs.

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1 Introduction

The existing literature on macroeconomic policy rules emphasizes the stabilizing properties of two alternative regimes: one of "monetary dominance"—whereby the central bank fights inflation aggressively and fiscal authorities stabilize public debt, and one of "fiscal dominance"—whereby fiscal authorities disregard debt stabilization and the central bank passively accommodates inflation taxes to rule out debt escalation (see, e.g., Leeper [1991], Taylor [1999], Woodford [2003], Canzoneri et al. [2010], Galí [2015], Leeper and Leith [2016], Bianchi et al. [2023], Cochrane [2023], Bordo et al. [2024]). A key aspect of policy design intended to ensure aggregate stability—it is well known—concerns shaping agents' expectations. For a stable macroeconomic environment largely relies on the ability of public authorities to account for how their future actions affect the expectations of the private sector (Anufriev et al. [2013]; Agliari et al. [2015]). A common characteristic of the studies extant that arrive at demonstrating the occurrence of equilibrium uniqueness and stability under either an active monetary-passive fiscal regime (monetary dominance) or a passive monetary-active fiscal regime (fiscal dominance)—the so-called Leeper [1991]'s dichotomy—is the assumption of forward looking, homogeneous agents with perfect foresight or rational expectations. In this paper, we subject the foregoing policy conclusions in terms of equilibrium determinacy to a more general test.

Specifically, our contribution is to study monetary-fiscal interactions in a New Keynesian model with boundedly rational agents and heterogeneous expectations. In 2010, during a speech, Jean-Claude Trichet, the then President of the European Central Bank, called for a new approach for policy makers to managing crises: "The atomistic, optimising agents underlying existing models do not capture behavior during a crisis period. We need to deal better with heterogeneity across agents and the interaction among those heterogeneous agents. [...] Very encouraging work is under way on new concepts, such as learning and rational inattention". Much research in behavioral economics and finance makes it increasingly evident that not all agents always act rationally. This evidence has led to the development of new models that account for the impact of emotions, biases, and other factors on market behavior. Although the importance of managing

expectations for conducting macroeconomic policy has widely been recognized and stressed—for example in Woodford and Walsh [2005], the question of how to manage expectations when forecasting rules are heterogeneous appears to be largely unexplored. Our study is an effort to fill this gap.

For our objectives, we set forth a New Keynesian model extended to incorporate both short-term and long-term bonds (as in Andrés et al. [2004]), along the lines recently proposed by Galì [2025], as well as financial frictions arising from adjustment costs that agents face whenever they change their portfolio allocation. This extension enables us to incorporate a term-structure dimension relative to the baseline New Keynesian model, whereby the financial structure is highly stylized, and assets are perfect substitutes. As a consequence, aggregate demand is influenced by a weighted average of the short-term interest rate and expectations about the future long-term interest rate. This feature appears to be particularly appropriate for our purposes, as it assigns a central role to agents' expectations not only about future inflation and the output gap, as in the standard setup, but also about the future path of long-term interest rates, which is directly shaped by monetary–fiscal interactions. Within this environment, agents form expectations using different forecasting rules. In particular, we adopt a heterogeneous expectations framework à la Brock and Hommes [1997], in which the ecology of forecasting rules evolves endogenously through a selection mechanism based on relative past performance, allowing agents to switch between forecasting strategies over time.

We use our model to characterize the resulting nonlinear equilibrium dynamics associated to the ecology of forecasting rules, and we find results that would not appear in the traditional New Keynesian framework. Specifically, our central finding is that neither the Taylor principle (Taylor [1993]; Bordo et al. [2024]) nor active fiscal policy in line with the fiscal theory of the price level (Woodford [1994, 1995]; Sims [1994]; Cochrane [1998, 2005, 2023]) guarantees uniqueness of equilibrium. Under both monetary and fiscal dominance, heterogeneous expectations easily open the door to aggregate instability in the form of either perpetual fluctuations around the initial steady state or attracting cycles that may potentially converge to a different steady state. Simulation

analysis applied to the calibrated economy demonstrates that the resulting oscillations are economically significant. It should be stressed that the existence of endogenous equilibrium cycles constitutes a severe case of monetary and fiscal policy-induced instability. This is because any trajectory originating in an arbitrarily small neighborhood around the steady state can be supported as an equilibrium outcome, hence giving rise to indeterminacy of equilibrium. This result is of particular interest, for it elucidates the fact that overlooking the heterogeneous way in which expectations are formed is not immaterial for the dynamic properties of monetary and fiscal policy rules. Bifurcation analysis, in particular, reveals that there typically exists a threshold value for the degree of switching between forecasting rules—on the basis of their past performance—above which equilibrium multiplicity in the form of endogenous cycles precisely prevails.

The paper also contributes to the quest for stabilizing policy rules. Our analysis indicates that a regime of monetary dominance whereby primary budget surpluses respond aggressively to debt accumulation is likely to ensure macroeconomic stability and insulate the economy from complex dynamics. This result clearly implies that in inflation targeting regimes supported by the application of the Taylor principle, the standard budgetary policy requirement according to which fiscal authorities should not engage in Ponzi's games and satisfy—in a Ricardian fashion—the intertemporal government's budget constraint for any possible path of macroeconomic variables is not sufficient to ensure a unique stable equilibrium. In an environment with heterogeneous expectations, a sufficiently strong fiscal policy reaction to increases in debt appears to be necessary for it has the effect of coordinating the evolutionary dynamics of adaptive beliefs, making cyclical equilibria unlikely to exist.

A final major result of our study is that the degree of financial frictions does matter for aggregate stability under both monetary and fiscal dominance. In particular, our simulation analysis demonstrates that *ceteris paribus* stronger financial frictions, by limiting arbitrage behavior, contribute to enlarge the determinacy region and mitigate the case for expectation-driven fluctuations.

The remainder of the paper is organized into eight sections. Section 2 elucidates the paper's

connections to the literature. Section 3 presents the model economy and describes how the term-structure dimension and the segmentation in the financial market is included in the New Keynesian framework. Sections 4-5 introduce heterogeneity across agents by means of an adaptive belief system. Sections 6-8 investigate the issue of macroeconomic stability under monetary and fiscal feedback policy rules and by means of simulations characterize the equilibrium properties. Finally, Section 9 provides concluding remarks.

2 Related Literature

Despite the fact that the forward looking representative agent paradigm is still the cornerstone of most contemporary macroeconomics, the awareness of its limitations is spreading among researchers, and even in mainstream macroeconomics the representative agent approach is not as eagerly embraced as in the past (Assenza and Berardi [2009]). For example, heterogeneous agent models mimic important observed stylized facts in asset returns, such as fat tails, clustered volatility and long memory, as discussed in the extensive surveys of LeBaron [2006] and Hommes [2006]. Some prominent examples of macro and financial models with heterogeneous expectations include Evans and Honkapohja [2003b, 2006], Bullard and Mitra [2002], Ascari et al. [2012], Hommes [2011], Assenza et al. [2013]. Experimental evidence from learning-to-forecast experiments shows that heterogeneous expectations generate fluctuations both in financial markets [Anufriev and Hommes, 2012] and in the real sector [Assenza et al., 2021]. These experiments underscore the importance of allowing for non-rational expectations and endogenous bubbles in macroeconomic models. Mankiw et al. [2003] show the disagreement among both consumers and professional economists in survey data about inflation expectations. Carroll [2003] emphasizes that information slowly diffuses through the economy and people only occasionally pay attention to news and this inattention generates "stickiness" in aggregate expectations with important macroeconomic consequences. Branch [2004, 2007] report that survey data about inflation expectations are consistent with a dynamic discrete choice between statistical predictor functions.

Departures from the standard New Keynesian model can change the equilibrium properties of the economy as documented in Bullard and Mitra [2002], Evans and Honkapohja [2003a,b] and Preston [2006] who show that, under adaptive learning, the unique rational expectations equilibrium may not be stable for some policy rules. Several works, for example Branch and McGough [2009, 2010] and De Grauwe [2011], introduce dynamic predictor selection into a New Keynesian model with heterogeneous expectations and examine its implications for monetary policy, finding that complex dynamics may arise even if the model under rational expectations has a unique stationary solution. Guerini [2013] investigates whether a Friedman k-percent monetary policy rule may be a valid instrument to counteract the instability created by the presence of heterogeneous expectations in an adaptive belief system. Furthermore, Anufriev et al. [2013] introduce heterogeneous expectations in a simple frictionless dynamic stochastic general equilibrium model to investigate inflation dynamics under alternative interest rate rules, and they find out that multiple steady states may arise even when the Taylor interest rate rule sets the interest rate more than point for point in response to inflation. More recently Anufriev et al. [2025] explore the "lean against the wind" policy efficiency in dampening the amplitude of endogenous fluctuations within the New Keynesian framework with heterogeneous expectations. Agliari et al. [2025] investigate the nonlinear dynamics and perform a bifurcation analysis in the context of a behavioral model for inflation with boundedly rational and heterogeneous expectations. Bullard et al. [2025] emphasize the destabilizing role of heterogeneous inflation expectations over the 2021-23 inflation surge and demonstrate the importance of of the monetary policy timing in containing inflation scares.

Our paper also builds on a growing literature that incorporates heuristic-based expectations into macroeconomic models. For example, Brazier et al. [2018] do so in the context of an overlapping generations model. Branch and Evans [2006] develop models in which agents must choose between misspecified models. Hence, although agents may have full information for several reasons, such as concerns about degrees of freedom, they may be fitting overly parsimonious models. Kurz and Motolese [2011] use models in which agents develop rational beliefs. In addition, there

is a large literature of behavioral finance models incorporating the view that agents are limited in their cognitive skills and use heuristics to guide their behavior and forecasts such as (Lux and Marchesi [2000] and De Grauwe and Grimaldi [2006]).

Our study differs from the foregoing literature in four important dimensions. First, in order to scrutinize the key role of heterogeneous expectations in shaping monetary-fiscal policy interactions, we lay out a New Keynesian model extended to incorporate a term-structure channel. In this way, we are able to examine the dynamic consequences of agents' expectations not only on the future path of inflation and the output gap, as in the standard theoretical setting considered by the existing literature, but also on the future path of the long-term interest rate, which is critically influenced by both the conduct of monetary and fiscal policy. Second, we characterize macroeconomic dynamics under heterogeneous expectations in an environment that embeds not only monetary policy feedback rules à la Taylor [1993, 1999], as the literature extant, but also fiscal policy feedback rules à la Leeper [1991] and Bohn [1995, 2008], distinguish between a regime of monetary dominance and a regime of fiscal dominance. Third, we add new insights about the quest for stabilizing policies when bounded rationality and an endogenous evolutionary selection of different forecasting strategies are operative. We show, in particular, that a regime of monetary dominance whereby fiscal authorities react in a sufficiently aggressive way to the accumulation of public debt is potentially capable of restoring the existence of a unique stable equilibrium and rule out multiple endogenous cycles. Forth, we are able to analyze the way in which financial frictions affect equilibrium dynamics and provide novel results about the stabilizing role driven by a relatively high degree of financial market segmentation.

From the latter perspective, the paper contributes to the literature of dynamic stochastic general equilibrium models with segmentation in financial markets. The existing studies, indeed, are mostly anchored to the fully rational representative agent paradigm. In particular, Andrés et al. [2004], Harrison [2012, 2017], Falagiarda [2014] and Marzo and Zagaglia [2018] introduce a term structure channel allowing to study its feedback to the economy. An important contribution of the paper precisely lies in examining the impact on model stability under monetary

and fiscal rules obtained when introducing the additional friction of bounded rationality and the evolutionary selection of individual expectations.

3 The Model

This section presents an otherwise standard New Keynesian model (as in Galí [2015]) extended to include segmentation in financial markets.

Consumers maximize utility, intermediate-good producers operate under monopolistic competition, and competitive firms produce the final good.

3.1 Households

Households' preferences are defined over consumption C_t and time devoted to market employment N_t, and each household maximizes the expected present discounted value of utility:

$$U_t = E_t \sum_{t=0}^{\infty} \beta^t u\Big(C_t, N_t\Big) \tag{1}$$

where β is the intertemporal discount factor. The utility function $u\Big(C_t, N_t\Big)$ is defined as follows:

$$u\Big(C_t, N_t\Big) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi} \tag{2}$$

where σ and ψ determine, respectively, the curvature of the utility of consumption (coefficient of relative risk aversion) and the disutility of labor.

In this economy, each agent i chooses the composition of a basket of differentiated final goods that enters the utility function. Preferences across varieties of goods are represented by a standard CES aggregator à la Dixit and Stiglitz [1977] of all the $j \in [0, 1]$ varieties of final goods produced in this economy, and it is defined as

$$C_t \equiv \left(\int_0^1 C_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \tag{3}$$

where C_t is the aggregate consumption index of all the differentiated final goods produced by each firm j and ϵ is the elasticity of substitution between different final goods varieties.

The associated aggregate price index for consumption is also defined as

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}. \tag{4}$$

Each agent solves an intertemporal portfolio allocation problem by maximizing the utility function subject to the following budget constraint, which features the two assets available in the economy:

$$P_t C_t + B_t + V_t \tilde{B}_t^L = R_{t-1} B_{t-1} + (1 + \chi V_t) \tilde{B}_{t-1}^L + W_t N_t + D_t - T_t \tag{5}$$

where W_t is the nominal wage, D_t are firms' dividends (profits), T_t are lump-sum taxes, B_t is a short-term bond and $\tilde{B}_t^L$ is a long-term bond. Long-term bonds are modeled following Woodford [2001] as infinite-maturity government bonds that pay a geometrically declining coupon [1]. In particular, a bond issued at date t pays nominal coupons χ^s at dates $t + 1 + s$, for $s \geq 0$. As a result, holding one dollar of a bond issued s periods ago is equivalent to holding χ^s dollars of a bond issued at date t. This representation allows us to express the household budget constraint in terms of a single bond price and a single stock of long-term bonds.

Households hold short-term bonds B_t, which pay a gross return R_t, as well as long-term

[1] Modeling long-term bonds as in Woodford [2001] provides a useful alternative to the assumptions adopted in Andrés et al. [2004] and Ljungqvist and Sargent [2004]. In those contributions, long-term bonds are modeled as zero-coupon, fixed-maturity instruments, and secondary markets for long-term bonds are absent. As a result, agents who purchase long-term government debt must hold it until maturity. These assumptions constitute a convenient simplification, as they reduce the number of state variables in the model. By contrast, modeling long-term bonds as infinite-maturity instruments paying a geometrically declining coupon allows them to be traded in every period. Consequently, optimal long-term bond holdings depend on the one-period return on bonds. This approach therefore represents an alternative but comparable simplification, preserving tractability while allowing for richer bond-trading dynamics.

bonds. Long-term bond holdings, denoted by $\tilde{B}_t^L$, are measured in units of newly issued bonds, and V_t denotes the nominal price of each bond. Holdings carried over from the previous period, $\tilde{B}_{t-1}^L$, yield one unit of currency and have a market value equal to χV_t. Hence, a quantity $\tilde{B}_{t-1}^L$ of bonds issued at date $t-1$ is equivalent to a quantity $\chi \tilde{B}_{t-1}^L$ of bonds issued at date t. This structure implies that long-term bonds can be traded each period and that optimal long-term bond holdings depend on the one-period return on bonds.

Following Harrison [2017], we write the household budget constraint in terms of the one-period return on long-term bonds by defining

$$B_t^L \equiv V_t \tilde{B}_t^L.$$

Under this formulation, the household's choice variable is the value of long-term bond holdings. Since households take bond prices as given, this representation is isomorphic to the original one, in the sense that solving the problem in terms of dollar investment in bonds or in terms of the number of bonds purchased yields identical equilibrium outcomes. The household budget constraint can be written as:

$$B_t^L + B_t = R_{t-1} B_{t-1} + B_{t-1}^L \frac{1 + \chi V_t}{V_{t-1}} + W_t N_t + D_t - T_t - P_t C_t. \tag{6}$$

Defining the one-period return on long-term bonds as

$$R_t^{1,L} \equiv \frac{1 + \chi V_t}{V_{t-1}}, \tag{7}$$

the budget constraint can be rewritten as

$$B_t^L + B_t = R_{t-1} B_{t-1} + R_t^{1,L} B_{t-1}^L + W_t N_t + D_t - T_t - P_t C_t. \tag{8}$$

Segmentation in financial markets is introduced through portfolio adjustment frictions, which

capture impediments to arbitrage activity that would otherwise equalize asset returns, as in Harrison [2017]. These adjustment costs are central to modeling bond-market segmentation, as they generate a wedge between short- and long-term interest rates. Accordingly, the household's maximization problem is subject to the budget constraint (8), augmented to include portfolio adjustment costs. Adjustment costs are specified as a function of deviations of the household's portfolio mix, $\frac{B_t}{B_t^L}$, from its desired level, δ^{-1}, and take the form [2]

$$AC = \frac{\Phi P_t(b + b^L)}{2}\left[\delta\frac{B_t}{B_t^L} - 1\right]^2 \tag{9}$$

where Φ captures the degree of imperfect substitutability between short- and long-term bonds, and hence the elasticity of long-term bond rates to variations in the portfolio composition. Following Andrés et al. [2004], δ is set equal to the steady-state ratio of long-term to short-term bonds, so that portfolio adjustment costs are zero at the non-stochastic steady state. As emphasized by Vayanos and Vila [2009], agents may have preferences over bond maturities, as in the preferred-habitat theory. This provides a natural justification for the introduction of portfolio frictions, since deviations from the preferred portfolio composition are costly to households.

These adjustment costs can also be interpreted as a proxy for households' exposure to liquidity risk. Longer-maturity bonds are typically less liquid and are therefore perceived as riskier than short-term assets. As a result, holdings of long-term bonds are associated with implicit liquidity costs, which induce households to partially offset these positions by holding additional short-term bonds. In this sense, agents endogenously impose precautionary liquidity holdings on their long-term investments, as in Andrés et al. [2004]. Finally, portfolio adjustment costs may also capture resources devoted to information acquisition or to the management of bond portfolios.

Taking these costs into account, the household budget constraint becomes

$$B_t^L + B_t = R_{t-1}B_{t-1} + R_t^{1,L}B_{t-1}^L + W_tN_t + D_t - T_t - P_tC_t - \frac{\Phi P_t(b + b^L)}{2}\left[\delta\frac{B_t}{B_t^L} - 1\right]^2 \tag{10}$$

[2] b and b^L denote the steady-state real levels of short-term and long-term bonds.

3.1.1 Optimality Conditions

Households maximize lifetime utility subject to the real budget constraint. The representative household's problem is solved using a Lagrangian formulation, with the full derivation reported in the Appendix A.1.

The first-order conditions with respect to consumption C_t, labor N_t, short-term bonds B_t and long-term bonds B_t^L are given by:

$$C_t^{-\sigma} = \lambda_t \tag{11}$$

$$\frac{W_t}{P_t} = N_t^{\psi} C_t^{\sigma} \tag{12}$$

$$-\lambda_t + \beta E_t \lambda_{t+1} R_t \Pi_{t+1}^{-1} - \lambda_t \Phi(b + b^L) \left[\delta \frac{B_t}{B_t^L} - 1 \right] \frac{\delta}{b_t^L} = 0 \tag{13}$$

$$-\lambda_t + \beta E_t \lambda_{t+1} R_{t+1}^L \Pi_{t+1}^{-1} - \lambda_t \Phi(b + b^L) \left[\delta \frac{B_t}{B_t^L} - 1 \right] \left[- \frac{\delta b_t}{(b_t^L)^2} \right] = 0 \tag{14}$$

where λ_t is the Lagrange multiplier.

3.2 Firms

The firms' sector follows a quite standard representation. Final good production is perfectly competitive, while intermediate goods are produced under monopolistic competition. Each intermediate firm j demands labor and fixes the price of its good given the following demand function:

$$C_{i,t} = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} C_t \tag{15}$$

A continuum of intermediate-good–producing firms, indexed by $j \in (0, 1)$, hire $N_t(j)$ units of labor from the representative household and produce $Y_t(j)$ units of the intermediate good

according to the following technology:

$$Y_t(j) = A_t N_t(j)^{1-\alpha} \tag{16}$$

with $(1-\alpha)$ the share of labor used in production and $\ln(A_t/A) = a_t$, where a_t denotes an exogenous technology (Total Factor Productivity) process evolving as

$$a_t = \rho_a a_{t-1} + \epsilon_t^a, \qquad \epsilon_t^a \sim \text{i.i.d. } N(0, \sigma^2_{\epsilon^a}). \tag{17}$$

Before setting prices, each firm chooses labor inputs so as to minimize total costs (TC_t):

$$\min_{N_t(j)} TC_t = W_t N_t(j) \tag{18}$$

subject to

$$Y_t(j) = A_t N_t(j)^{1-\alpha}. \tag{19}$$

The first-order condition with respect to $N_t(j)$ is given by

$$\frac{W_t}{A_t} = (1-\alpha) N_t(j)^{-\alpha} MC_t(j), \tag{20}$$

where $MC_t(j)$ denotes marginal costs and corresponds to the Lagrange multiplier of the cost minimization problem. Firms then choose prices to maximize profits:

$$\max_{P_t^*} P_t^* Y_t(j) - MC_t(j) Y_t(j) \tag{21}$$

subject to

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t. \tag{22}$$

The optimal price is

$$P_t^* = \frac{\epsilon}{\epsilon - 1} MC_t(j), \tag{23}$$

where $\frac{\epsilon}{\epsilon-1} > 1$ defines the flexible-price markup under monopolistic competition.

Each firm sets an optimal price P_t^*, but prices are sticky. Following Calvo [1983], in each period, firms are allowed to reset their price with probability $(1-\theta)$, while with probability θ they keep their price unchanged at the level set in the previous period. Accordingly, θ denotes the fraction of firms that do not adjust prices, and $1-\theta$ the fraction that reoptimize.
Reoptimizing firms choose the price P_t^* to maximize the expected discounted sum of future profits. As a result, the aggregate price level evolves according to

$$P_t = \left[(1-\theta)(P_t^*)^{1-\epsilon} + \theta(P_{t-1})^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}, \tag{24}$$

which corresponds to a weighted average of the newly set optimal price and the lagged price, consistent with the CES price index. By combining the equations characterizing firms' price-setting behavior, it is possible to obtain the final price equation.

3.3 The Government Sector

The government issues two types of bonds: short-term and long-term bonds. Given the specification of long-term bonds described above (Woodford [2001]), the nominal government budget constraint is given by

$$B_t + V_t \tilde{B}_t^L = R_{t-1} B_{t-1} + (1 + \chi V_t)\tilde{B}_{t-1}^L - T_t. \tag{25}$$

There is no government spending, and net taxes are lump-sum, so that taxation does not distort private economic decisions.

3.4 The Log-linearized Model

This section derives the log-linearized version of the model in order to reduce the system of equilibrium conditions and obtain a reduced-form representation. To this end, the model is log-

linearized around the non-stochastic steady state. Lowercase letters denote nominal quantities deflated by the price index, and the notation $\hat{x}_t = \ln \frac{x_t}{x}$ denotes the log deviation of variable x_t from its steady-state value x.

The household's first-order conditions can be expressed in log-linear form and combined to yield

$$\hat{w}_t = \psi \hat{n}_t + \sigma \hat{c}_t, \tag{26}$$

$$E_t \hat{R}^{1,L}_{t+1} = \hat{R}_t - \Phi(1+\delta)^2(\hat{b}_t - \hat{b}^L_t), \tag{27}$$

$$\hat{c}_t = E_t \hat{c}_{t+1} - \frac{1}{\sigma}\left(\frac{1}{1+\delta}\hat{R}_t - E_t\hat{\pi}_{t+1} + \frac{\delta}{1+\delta}E_t\hat{R}^{1,L}_{t+1}\right). \tag{28}$$

Throughout, $\hat{x}_t = \log x_t - \log x$ denotes the log deviation of variable x_t from its steady-state value. Equation (26) characterizes labor supply, Equation (27) is the term-structure equation obtained by combining the log-linearized versions of Equations (13) and (14), and Equation (28) represents the Euler equation. Equation (28) shows that aggregate demand is driven by a weighted average of short-term and long-term interest rates. In addition, aggregate demand depends on households' relative holdings of short-term and long-term bonds, as captured by Equation (27). An increase in the relative holdings of short-term bonds acts as a reduction in the short-term real interest rate and stimulates aggregate demand. Following the interpretation in Andrés et al. [2004], an increase in relative short-term bond holdings reflects an expansion in households' marginal liquidity. As a result, a shift toward short-term bonds reduces the wedge between long-term and short-term rates of return, as shown in Equation (27).

From the supply side, by log-linearizing and combining the equations governing price setting, we obtain the following log-linear price equation:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \xi \widehat{mc}_t. \tag{29}$$

where ξ is a composite parameter that depends on structural parameters, as in the textbook New Keynesian model of Galí [2015].

The model can be conveniently expressed in terms of the output gap, defined as $\hat{x}_t = \hat{y}_t - \hat{y}_t^n$. Taking into account that $\hat{c}_t = \hat{y}_t$ and $\hat{c}_t^n = \hat{y}_t^n$, the IS curve can be rewritten in terms of output deviations from its natural level as

$$\hat{x}_t = E_t\hat{x}_{t+1} - \frac{1}{\sigma}\left(\frac{1}{1+\delta}\hat{R}_t - E_t\hat{\pi}_{t+1} - r_t^n + \frac{\delta}{1+\delta}E_t\hat{R}_{t+1}^{1,L}\right). \tag{30}$$

Finally, substituting for $\widehat{mc}_t$ into Equation (29) through well-known manipulations yields the price-setting equation, namely the New Keynesian Phillips Curve (NKPC):

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \kappa\hat{x}_t. \tag{31}$$

with $\kappa = \xi\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)$.

3.5 Central Bank

We assume that the central bank controls the nominal short-term interest rate by setting a policy rule that governs the adjustment of its policy instrument:

$$\hat{R}_t = \phi_\pi\hat{\pi}_t + \phi_x\hat{x}_t + v_t. \tag{32}$$

The monetary policy shock follows the process

$$v_t = \rho_v v_{t-1} + \epsilon_t^v, \qquad \epsilon_t^v \sim i.i.d.\, N(0, \sigma_{\epsilon_t^v}^2)$$

According to Equation (32), the nominal short-term interest rate responds contemporaneously to fluctuations in current inflation and the output gap.

3.6 Fiscal Policy

We log-linearize the real government budget constraint, around the steady state, which yields:

$$\frac{1}{1+\delta}(\hat{b}_t + \delta\hat{b}_t^L) = \frac{1}{1+\delta}\beta^{-1}(\hat{b}_{t-1} + \hat{R}_{t-1}) + \frac{\delta}{1+\delta}\beta^{-1}(\hat{b}_{t-1}^L + \hat{R}_t^L) - \beta^{-1}\hat{\pi}_t - \frac{\tau}{b+b^L}\hat{t}_t \quad (33)$$

where τ denotes the steady-state level of T_t. As emphasized by Leeper [1991] and Sims [1994], to prevent a persistent and unbounded increase in prices it is necessary to introduce policy rules under which taxes respond to changes in the outstanding level of real public debt. In other words, taxes can not evolve independently of the stock of government liabilities outstanding in the economy. We consider a fiscal rule that not only targets the lagged stock of public debt to stabilize the debt issuance process, but also adjusts household payments so as to offset the financing costs associated with previously issued short-term debt, as in Harrison [2012].

$$\frac{\tau}{b+b^L}\hat{t}_t = \beta^{-1}\hat{R}_{t-1} + \zeta\hat{b}_{t-1}. \quad (34)$$

The rule is specified in log-linear form. The parameter ζ measures the elasticity of household payments with respect to changes in the outstanding stock of short-term debt.

Finally, for simplicity, the supply of long-term bonds is assumed to follow an exogenous autoregressive process, as in Andrés et al. [2004] and Falagiarda [2014]:

$$\hat{b}_t^L = \rho_{b^L}\hat{b}_{t-1}^L + \epsilon_{b^L}, \qquad \epsilon_{b^L} \sim i.i.d.\mathcal{N}(0, \sigma^2_{\epsilon_t^{b^L}}) \quad (35)$$

3.7 Reduced-Form Representation

The model can be reduced to a system of five equations and written in state-space form, which is convenient for simulation and for studying the model's dynamic properties. Hereafter, log-linearized variables are simply denoted by lowercase letters, and the one-period long-term interest rate is denoted by R_t^L. Appendix A.2 provides further details on the derivation of the system and the expressions for the model coefficients.

The model in state-space form is given by

$$\begin{pmatrix} x_t \\ \pi_t \\ R_t^L \\ b_t \\ R_t \end{pmatrix} = r \begin{pmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 \\ a_{41} & a_{42} & a_{43} & 0 & 0 \\ a_{51} & a_{52} & a_{53} & 0 & 0 \end{pmatrix} \begin{pmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \\ E_t R_{t+1}^L \\ E_t b_{t+1} \\ E_t R_{t+1} \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_{34} & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_{t-1} \\ \pi_{t-1} \\ R_{t-1}^L \\ b_{t-1} \\ R_{t-1} \end{pmatrix} + M. \tag{36}$$

The Appendix A.2 reports the expressions for the model coefficients.

4 The Structure of Heterogeneous Expectations

In this section, we specify the expectations formation mechanism by presenting the adaptive belief system by Brock and Hommes [1997] that allows to model expectations by introducing heterogeneity among agents. People could be different in some aspects (e.g., degree of rationality, information set, etc.) and heterogeneity is an important feature of any economic story. If people want to pursue more optimal decision rules, they must face some search costs; due to these limitations, agents are endowed only with a bounded rationality and hence in order to make forecasts or to take a decision that entails a degree of uncertainty, they use simple heuristics and simple rules which lead to biased predictions (Assenza and Berardi [2009]).

The adaptive learning mechanism allows individuals to choose between heuristics according to a 'fitness' criterion; individuals evaluate their past predictions and decide whether to select a certain heuristic or switching to the other one. Let us assume that agents can form expectations by choosing from H different forecasting strategies; we denote with $\hat{E}_{h,t} x_{t+1}$, $\hat{E}_{h,t} \pi_{t+1}$, $\hat{E}_{h,t} R_{t+1}^L$ the forecasts of output, inflation and long-term interest rate for an agent who has chosen the forecasting strategy h from H. Moreover, each rule for output, for inflation and long-term interest rate prediction can be chosen by different number of individuals: therefore the fraction of agents using forecasting rule h to forecast output at time t is denoted by $n_{h,t}^x$, the one for inflation $n_{h,t}^\pi$ and the one for long-term interest rate $n_{h,t}^{R^L}$.

The beliefs on output gap, inflation and long-term interest rate are updated over time, and as a consequence also the fractions of agents using a particular rule h are updated, according to an evolutionary fitness measure. The fitness measures for output, inflation and for interest rate respectively are publicly available but subject to noises and they are expressed in utility terms as:

$$\tilde{U}_{h,t} = U_{h,t} + \epsilon_{h,i,t} \tag{37}$$

$$\tilde{V}_{h,t} = V_{h,t} + \epsilon_{h,i,t} \tag{38}$$

$$\tilde{Z}_{h,t} = Z_{h,t} + \epsilon_{h,i,t} \tag{39}$$

where $U_{h,t}$, $V_{h,t}$ and $Z_{h,t}$ are the deterministic parts (we assume they can be red in a freely available newspaper) and $\epsilon_{h,i,t}$ is the stochastic component of the fitness measures. It collects independent and identically distributed noises, idiosyncratic across each time t, each strategy $h = 1, 2, ...H$ and each agent i.

A theorem on discrete choice by Manski and McFadden [1981] states that, assuming that the noises $\epsilon_{h,i,t}$ are drawn from a double exponential distribution, as the number of agents i goes to infinity the probability that an agent picks the strategy h is given by the discrete choice fractions[3]. The fraction of agents choosing strategy h is given by the well known discrete choice model:

$$n^{x}_{h,t} = \frac{e^{\gamma U_{h,t-1}}}{\sum_{h=1}^{H} e^{\gamma U_{h,t-1}}} \tag{40}$$

$$n^{\pi}_{h,t} = \frac{e^{\gamma V_{h,t-1}}}{\sum_{h=1}^{H} e^{\gamma V_{h,t-1}}} \tag{41}$$

[3]The framework developed in Matějka and McKay [2015] derives the probabilistic choice between predictors as the outcome of an optimization problem under rational inattention with discrete and symmetric options, and the main finding is that the decision maker's optimal information processing strategy results in probabilistic choices that follow a logit model.

$$n_{h,t}^{R^L} = \frac{e^{\gamma Z_{h,t-1}}}{\sum_{h=1}^{H} e^{\gamma Z_{h,t-1}}} \tag{42}$$

with γ the intensity of choice parameter that reflects the sensitivity of agents in selecting the optimal strategy. Notice that the higher the fitness of a forecasting rule h, the higher the probability that an agent will select strategy h. The functions $U(\cdot)$, $V(\cdot)$ and $Z(\cdot)$ are the past squared forecast errors[4], as in Anufriev et al. [2013]

$$U_{h,t-1} = -(x_{t-1} - \hat{E}_{h,t-2} x_{t-1})^2 \tag{43}$$

$$V_{h,t-1} = -(\pi_{t-1} - \hat{E}_{h,t-2} \pi_{t-1})^2 \tag{44}$$

$$Z_{h,t-1} = -(R_{t-1}^L - \hat{E}_{h,t-2} R_{t-1}^L)^2 \tag{45}$$

The stochastic process $\epsilon_{h,i,t}$ is reflected into the parameter $\gamma \in [0, \infty)$ and it is inversely related to the standard deviation of σ_ϵ. When $\gamma = 0$, $\sigma_\epsilon = 1$, meaning that differences across the h fitness measures $U_{h,t-1}$, $V_{h,t-1}$ and $Z_{h,t-1}$ can not be observed, hence the agent does not switch between strategies and the fractions are all constant and equal to $1/H$. Oppositely, $\gamma = \infty$ implies $\sigma_\epsilon = 0$ and this is the case where no noise is present, the fitness measure for each strategy is perfectly observable and in each time all agents pick the forecasting rule with the higher performance in the previous period.

5 Evolutionary Dynamics with Few Constant Belief Types

We consider a scenario in which agents can choose among $H + 1 (H = 0, 1, 2, ..., h)$, H different symmetric forecasting rules, where the positive and negative biases are exactly balanced around the target equilibrium. This choice implies that the targeted equilibrium is among the steady

[4] De Grauwe [2011] considers this willingness to learn via continuous evaluation of individual performance as "the most fundamental definition of rational behavior".

states of the model. Thus, the correct belief about the target is

$$\hat{E}_{0,t}x_{t+1} = \hat{E}_{0,t}\pi_{t+1} = \hat{E}_{0,t}R^L_{t+1} = 0 \tag{46}$$

As a first step, for the simulations, we consider the simplest scenario in which agents can choose between three different forecasting rules, available to the agents for output gap, inflation and interest rate forecasts, i.e. fundamental (the target equilibrium), optimistic and pessimistic predictors:

$$\hat{E}_{1,t}x_{t+1} = 0, \quad \hat{E}_{2,t}x_{t+1} = b, \quad \hat{E}_{3,t}x_{t+1} = -b$$

$$\hat{E}_{1,t}\pi_{t+1} = 0, \quad \hat{E}_{2,t}\pi_{t+1} = d, \quad \hat{E}_{3,t}\pi_{t+1} = -d$$

$$\hat{E}_{1,t}R^L_{t+1} = 0, \quad \hat{E}_{2,t}R^L_{t+1} = f, \quad \hat{E}_{3,t}R^L_{t+1} = -f$$

with bias parameters $d > 0$, $b > 0$ and $f > 0$.

The market forecast is obtained as a weighted average of the H predictors:

$$\tilde{E}_t x_{t+1} = \sum_{h=0}^{H} n^x_{h,t}\hat{E}_{h,t}x_{t+1} \tag{47}$$

$$\tilde{E}_t \pi_{t+1} = \sum_{h=0}^{H} n^\pi_{h,t}\hat{E}_{h,t}\pi_{t+1} \tag{48}$$

$$\tilde{E}_t R^L_{t+1} = \sum_{h=0}^{H} n^{R^L}_{h,t}\hat{E}_{h,t}R^L_{t+1} \tag{49}$$

Substituting (37), (40) in (47) and (38), (41) in (48) and (39), (42) in (49), and taking also into

account that the rational agents have predictor (46), we get:

$$\tilde{E}_t x_{t+1} = \frac{\sum_{h=1}^{H} b_h e^{-\gamma(x_{t-1}-b_h)^2}}{\sum_{h=1}^{H} e^{-\gamma(x_{t-1}-b_h)^2} + e^{-\gamma(x_{t-1})^2+C}} \tag{50}$$

$$\tilde{E}_t \pi_{t+1} = \frac{\sum_{h=1}^{H} d_h e^{-\gamma(\pi_{t-1}-d_h)^2}}{\sum_{h=1}^{H} e^{-\gamma(\pi_{t-1}-d_h)^2} + e^{-\gamma(\pi_{t-1})^2+C}} \tag{51}$$

$$\tilde{E}_t R^L_{t+1} = \frac{\sum_{h=1}^{H} f_h e^{-\gamma(R^L_{t-1}-f_h)^2}}{\sum_{h=1}^{H} e^{-\gamma(R^L_{t-1}-f_h)^2} + e^{-\gamma(R^L_{t-1})^2+C}} \tag{52}$$

Now by inserting (50), (51) and (52) into the system (36), it is possible to find system that we want to analyze for the stability analysis. The expected value operator with the tilde denotes a convex combination of heterogeneous expectations. There is a hidden non-linearity in the system, due to the fact that the weights of the convex combination of heuristics that yield the average expectations are themselves non-linear functions of the lagged, current and expected inflation, output gap and interest rate.

In this approach, it is crucial to address a methodological issue: the forecasting rules introduced here are not derived from micro-level foundations and then aggregated. Instead, they are applied ex post. As noted by De Grauwe [2011], this methodology aligns with the learning literature initiated by Evans et al. [2001]. Branch and McGough [2009] propose a framework that requires a specific set of axioms for aggregation. These axioms impose regularity on the expectation operator, facilitating the aggregation process. Notably, they emphasize that their axioms ensure agents' expectations comply with the Law of Iterated Expectations (LIE) at both individual and aggregate levels.

Moreover, Kurz et al. [2013] specifies rationality axioms on beliefs, rather than on expectation operators, presenting a more natural pathway to a New Keynesian model characterized by diverse beliefs. This framework allows for aggregation in the log-linearized economy. Hence, within this setting, it becomes feasible to aggregate the first-order conditions of agents and average each firm's pricing decisions—derived under subjective expectations—to formulate structural

equations consistent with heterogeneous beliefs; this represents a first-order approximation to general nonlinear aggregation of heterogeneous expectations. Indeed, the model used in this paper can be considered as a slightly simplified version of their model derived under the classical assumptions. Furthermore, the aggregate demand equation in the model can be derived using the Euler learning approach outlined by Hommes and Lustenhouwer [2019], ensuring consistency with the aggregate equation derived above.

Inspection of system (36) indicates that deriving general analytical results on stability is difficult. Accordingly, we study the dynamic behavior of the system using numerical methods. For the numerical analysis, we employ the calibration reported in Table 1.

Table 1: Baseline Parameter Calibration

Parameter	Description	Value
σ	Coefficient of relative risk aversion	1
β	Discount factor	0.99
θ	Calvo probability of price non-adjustment	0.75
ψ	Inverse labor supply elasticity	1
α	Share of capital in the production function	0.33
κ	Slope coefficient (NKPC)	0.0863
ϵ	Elasticity of substitution across varieties	4
δ	Steady-state ratio $\frac{b^L}{b}$	1.53
Φ	Elasticity of the long-term bond rate w.r.t. portfolio mix	0.09
b	Bias parameter for output gap	1
d	Bias parameter for inflation	1
f	Bias parameter for interest rate	1

The key parameter governing aggregate demand, as well as the discount factor, are set to standard values commonly used in the literature. The steady-state ratio of long-term to short-term bonds, δ, is set to 1.53, in line with the estimates of the shares of short-term and long-term debt in total public debt reported by Falagiarda [2014], based on U.S. data. The parameter governing portfolio adjustment costs is set to $\Phi = 0.09$, following Harrison [2012].

Finally, policy parameters are chosen to allow for the analysis of both monetary dominance and fiscal dominance regimes, highlighting how the interaction between monetary and fiscal policy affects the evolution of the system over time. In the spirit of Leeper [1991], we consider

different parameter configurations and examine how they affect equilibrium properties.

In the monetary policy rule, the response to the output gap is fixed at $\phi_x = 0.125$, while the response to inflation varies across regimes. Under monetary dominance, monetary policy satisfies the Taylor principle with $\phi_\pi = 1.5$, whereas under fiscal dominance the response to inflation is weaker (limit case $\phi_\pi = 0$). On the fiscal side, the feedback parameter in the tax rule is set to a passive baseline value of $\zeta = 0.25$. As a sensitivity analysis, we also consider alternative values of ζ, corresponding to a more aggressive fiscal response to fluctuations in the short-term debt stock. The baseline values of the policy parameters across regimes are summarized in Table 2.

Table 2: Policy Parameters Across Regimes

Parameter	Monetary Dominance	Fiscal Dominance
ϕ_π	1.5	0
ζ	0.25	0

6 Equilibrium Dynamics under Monetary Dominance

We start from a monetary dominance regime and focus on the three forward-looking variables of interest, i.e. output gap, inflation, and the long-term interest rate. As shown in Figure 1, for intermediate levels of agents' sensitivity, the resulting time series exhibit a 2-cycle dynamic pattern.

To gain further insight into the sources of dynamic instability, we investigate how the system's behavior changes as the parameter γ varies, that is, as the intensity-of-choice parameter governing the degree of nonlinearity in the model changes. To examine the relationship between parameter values and stability, we present the bifurcation diagram for our variable of interest, i.e. the long-term interest rate R_t^L as γ varies. In addition, we complement the bifurcation analysis with an examination of the largest Lyapunov exponent, which measures the average exponential rate of divergence of nearby trajectories in the dynamical system. A positive Lyapunov exponent indicates chaotic dynamics and sensitivity to initial conditions, whereas a negative Lyapunov

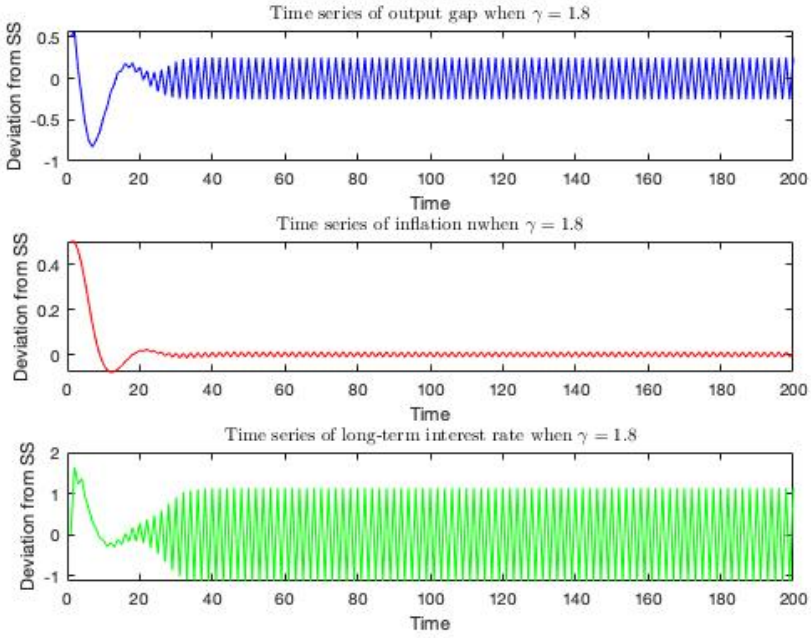


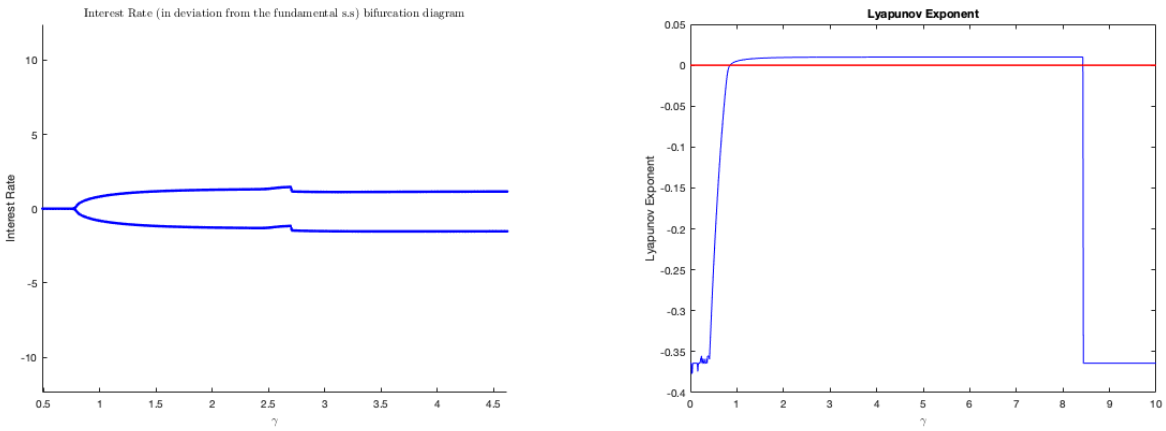
Figure 1: Time series of the output gap, inflation, and long-term interest rate under monetary dominance ($\phi_\pi = 1.5$) and passive fiscal policy ($\zeta = 0.25$).

exponent is associated with stable dynamics.

The bifurcation diagram in Figure 2 indicates that the system exhibits a bifurcation at approximately $\gamma \approx 0.8$. For values in the range $0.8 < \gamma < 8.3$, the largest Lyapunov exponent is slightly positive, suggesting that although a stable two-cycle emerges, the dynamics in its neighborhood may display sensitivity to initial conditions.

In order to fully characterize the bifurcation diagram we compute and plot the absolute eigenvalues of the Jacobian matrix of the following system of equations:

$$
\begin{cases}
x_t = r\sigma(1+\delta)\hat{E}_t x_{t+1} + r\big[(1+\delta) - \beta\phi_\pi\big]\hat{E}_t\pi_{t+1} - r\delta\hat{E}_t R^L_{t+1} \\
\pi_t = r\sigma\kappa(1+\delta)\hat{E}_t x_{t+1} + r\big[\beta(\sigma+\phi_x+\delta\sigma) + \kappa(1+\delta)\big]\hat{E}_t\pi_{t+1} - \delta r\kappa\hat{E}_t R^L_{t+1} \\
R^L_t = ra_{31}\hat{E}_t x_{t+1} + ra_{32}\hat{E}_t\pi_{t+1} - ra_{33}\hat{E}_t R^L_{t+1} + \big[\beta\zeta(1+\delta) - 1\big]\delta^{-1}b_{t-1} + R_{t-1} \\
b_t = r\dfrac{\sigma(\phi_x+\kappa\phi_\pi)}{\Phi(1+\delta)}\hat{E}_t x_{t+1} + r\dfrac{\phi_x+\kappa\phi_\pi+\beta\phi_\pi\sigma}{\Phi(1+\delta)}\hat{E}_t\pi_{t+1} - r\dfrac{\delta(\phi_x+\kappa\phi_\pi)+[\sigma(1+\delta)+\phi_x+\kappa\phi_\pi]}{\Phi(1+\delta)^2\big[\sigma(1+\delta)+\phi_x+\kappa\phi_\pi\big]}\hat{E}_t R^L_{t+1} \\
R_t = r\sigma(\phi_x+\kappa\phi_\pi)(1+\delta)\hat{E}_t x_{t+1} + r[\beta(\phi_\pi\sigma+\delta\phi_\pi\sigma) + (\phi_x+\kappa\phi_\pi)(1+\delta)]\hat{E}_t\pi_{t+1} - r\delta(\phi_x+\kappa\phi_\pi)\hat{E}_t R^L_{t+1}
\end{cases}
\tag{53}
$$



(a) Bifurcation Diagram for R_t^L **(b) Lyapunov Exponent**

Figure 2: Monetary dominance w $\zeta = 0.25$

After the substitution of equations (50), (51) and (52) in the system of equations (53) (as shown in the Appendix A.2), we compute the Jacobian matrix at the target equilibrium:

$$J = r \begin{pmatrix} \frac{4\gamma\sigma(1+\delta)}{2+e^{\gamma}} & \frac{4\gamma(1+\delta)-\beta\phi_{\pi}}{2+e^{\gamma}} & -\frac{4\gamma\delta}{2+e^{\gamma}} & 0 & 0 \\ \frac{4\gamma\sigma\kappa(1+\delta)}{2+e^{\gamma}} & \frac{4\gamma\beta(\sigma+\phi_x+\delta\sigma)+\kappa(1+\delta)}{2+e^{\gamma}} & -\frac{4\gamma\delta\kappa}{2+e^{\gamma}} & 0 & 0 \\ \frac{4\gamma a_{31}}{2+e^{\gamma}} & \frac{4\gamma a_{33}}{2+e^{\gamma}} & \frac{4\gamma a_{33}}{2+e^{\gamma}} & \frac{[\beta\zeta(1+\delta)-1]}{\delta} & 1 \\ \frac{4\gamma[\sigma(\phi_x+\kappa\phi_{\pi})]}{2+e^{\gamma}[\Phi(1+\delta)]} & \frac{4\gamma[\phi_x+\kappa\phi_{\pi}+\beta\phi_{\pi}\sigma]}{2+e^{\gamma}[\Phi(1+\delta)]} & -\frac{4\gamma[\delta(\phi_x+\kappa\phi_{\pi})+[\sigma(1+\delta)+\phi_x+\kappa\phi_{\pi}])}{2+e^{\gamma}\Phi(1+\delta)^2[\sigma(1+\delta)+\phi_x+\kappa\phi_{\pi}]} & 0 & 0 \\ \frac{4\gamma[\sigma(\phi_x+\kappa\phi_{\pi})(1+\delta)]}{2+e^{\gamma}} & \frac{4\gamma[(\phi_x+\kappa\phi_{\pi})(1+\delta)+\beta(\phi_{\pi}\sigma+\delta\phi_{\pi}\sigma)]}{2+e^{\gamma}} & -\frac{4\gamma[\delta(\phi_x+\kappa\phi_{\pi})]}{2+e^{\gamma}} & 0 & 0 \end{pmatrix}$$

We plot in Figure 3 the absolute eigenvalues $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ of the Jacobian matrix.

For $\gamma < 0.8$, all eigenvalues lie inside the unit circle, indicating that the system is stable. At $\gamma = 0.8$, one eigenvalue crosses the unit circle and becomes larger than one in modulus ($|\lambda_1| > 1$). According to nonlinear dynamics theory, this behavior is characteristic of a period-doubling bifurcation. In particular, the crossing of the unit circle implies that the fixed point loses stability and a stable periodic orbit (a two-cycle) emerges. It is important to note that for values of $\gamma > 2$, the system is locally stable, as confirmed by the eigenvalues. In this case, convergence to the fundamental steady state occurs only for initial conditions sufficiently close to the steady

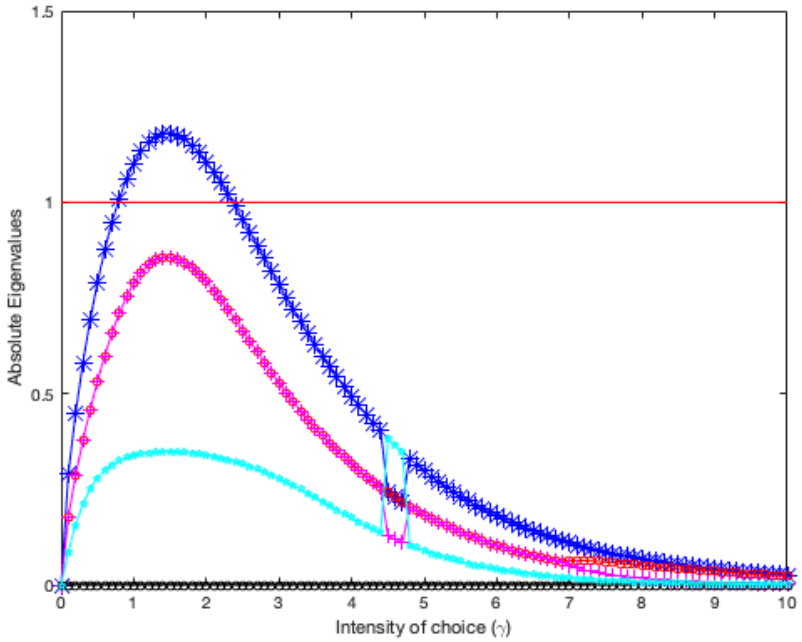


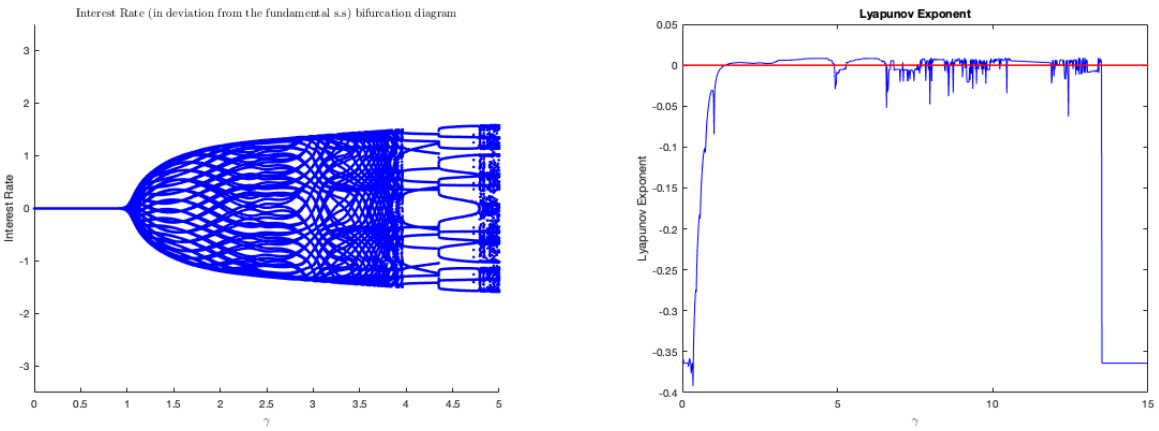
Figure 3: Representation of the absolute eigenvalues as a function of the intensity of choice parameter ($\zeta = 0.25$)

state. For initial conditions that deviate even slightly further away, convergence fails, indicating that stability is local rather than global.

We now consider a more responsive fiscal policy by increasing the calibration of the fiscal feedback parameter to $\zeta = 0.75$, in order to assess whether and how the stability properties of the system change.

Figure 4 shows that the dynamics become more complex for certain values of the intensity-of-choice parameter. The bifurcation diagram in Figure 4 reveals distinct regimes. For $\gamma \in (0, 1]$, that is, when the intensity of choice is relatively low, the diagram collapses to a single line, indicating stability and convergence to the fundamental steady state. For $\gamma > 1$, more complex dynamics emerge.

The Lyapunov exponent analysis provides further insight into the nature of these dynamics. As shown in Figure 4 (b), three regions can be identified. Two of them are associated with stable dynamics, as the Lyapunov exponent is significantly below zero, while the remaining region—where the Lyapunov exponent fluctuates around and above zero—suggests the emergence of



(a) Bifurcation Diagram for R_t^L **(b) Lyapunov Exponent**

Figure 4: Monetary dominance with $\zeta = 0.75$

quasi-periodic dynamics. When γ is relatively low or relatively high, the Lyapunov exponent is negative, implying stability of the system. At $\gamma = 1$, a bifurcation occurs, and for $\gamma \in (1, 13.5)$ the system exhibits sensitivity to initial conditions, indicating that chaotic dynamics may arise.

To characterize the bifurcation, we compute the eigenvalues of the Jacobian matrix associated with system (53) and plot their moduli as a function of the intensity-of-choice parameter. Figure 5 reports the absolute values of the five eigenvalues, $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$, as γ varies.

For $\gamma < 1$, all eigenvalues lie inside the unit circle, indicating that the system is locally stable. At the critical value $\gamma = 1$, a pair of complex conjugate eigenvalues crosses the unit circle, leading to a loss of stability. This pattern is consistent with the occurrence of a Neimark–Sacker bifurcation. At this point, the steady state loses stability and a periodic orbit emerges, signaling the transition to a more complex dynamical regime. This behavior is consistent with the results discussed above. Although the eigenvalues return inside the unit circle for $\gamma > 2$, suggesting local stability, the bifurcation diagram indicates the persistence of complex dynamics. This apparent discrepancy arises because eigenvalue analysis characterizes only local stability. In particular, for $1 < \gamma < 2$, the system is unstable for all initial conditions. For $\gamma > 2$, stability is local and convergence to the steady state occurs only for initial conditions sufficiently close to the fundamental steady state. For slightly larger deviations, the system may enter quasi-periodic dynamics, as

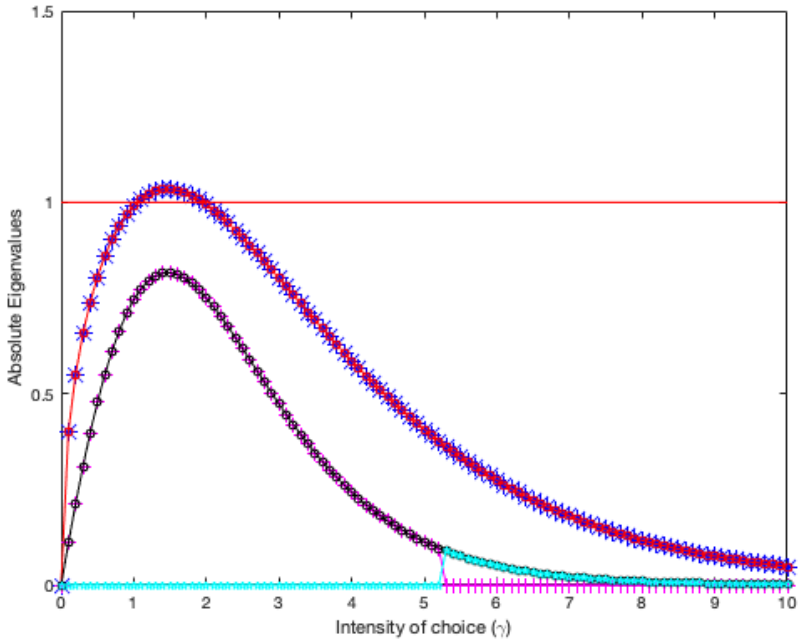


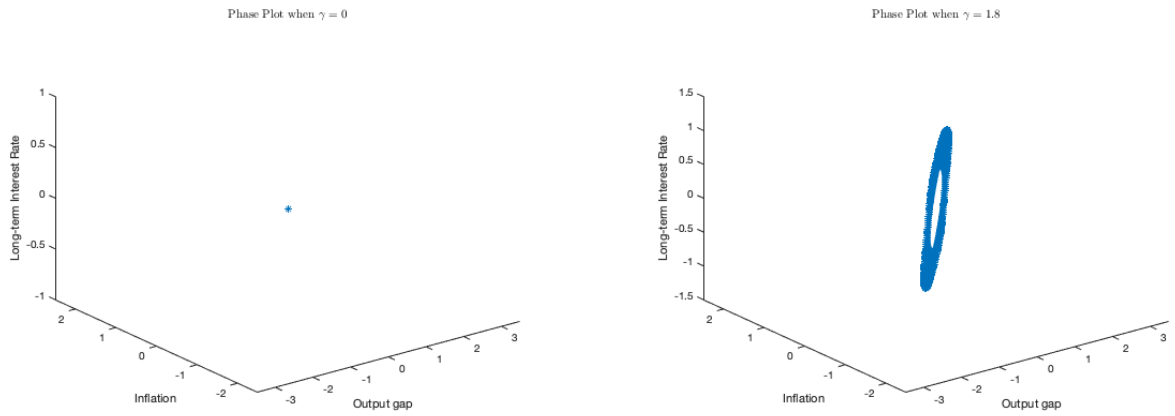
Figure 5: Absolute eigenvalues of the Jacobian matrix as a function of the intensity-of-choice parameter ($\zeta = 0.75$).

illustrated by the bifurcation diagram. We therefore conclude that for relatively low values of the intensity-of-choice parameter, the system is stable and financial market segmentation does not generate complex dynamics. By contrast, for intermediate or relatively high values of γ, stronger switching behavior by agents may give rise to quasi-periodic dynamics and to a multiplicity of equilibria.

Figure 6 consists of two panels, each displaying the phase plot of inflation, the output gap, and the long-term interest rate for different values of the intensity-of-choice parameter. The figure confirms the results discussed above. When the intensity of choice is low ($\gamma = 0$), output, inflation, and the long-term interest rate converge to their steady-state values. In this case, the attractor collapses to a single point at $(0, 0, 0)$ in phase space (left panel), indicating zero deviations from the fundamental equilibrium [5]. By contrast, when the intensity of choice increases

[5]Another configuration characterized by convergence to the fundamental steady state arises when γ is high, so that agents in the economy have perfect knowledge of predictors' performance and immediately switch to the predictor with the lowest forecasting error in the previous period. In this case, convergence may occur only if the system starts sufficiently close to the fundamental steady state, as shown in the Appendix A.3

to an intermediate level and exceeds the critical threshold, the system exhibits quasi-periodic dynamics characterized by the emergence of an invariant circle.



(a) **(b)**

Figure 6: Phase Plot when $\gamma = 0$ and $\gamma = 1.8$ ($\zeta = 0.75$)

So far, the analysis has focused on two configurations in which the system exhibits instability. However, the presence of instability is not a general feature of the monetary dominance regime. In fact, there exist parameter configurations under monetary dominance for which the system converges to the fundamental steady state. These cases highlight that instability arises only for specific combinations of behavioral intensity and policy parameters, while alternative configurations are compatible with stable dynamics.

When the fiscal response is increased to intermediate levels $\zeta > 0.33$, the feedback from debt becomes sufficiently mitigated to allow expectations to stabilize, as shown in Figure 7. In this case, the interaction between debt dynamics, long-term interest rate, and agents' expectations becomes self-correcting rather than destabilizing, and the system converges to the targeted equilibrium. However, as discussed above, when the fiscal response parameter increases and exceeds approximately 0.75, the system becomes unstable again. An excessively strong fiscal reaction to debt induces large adjustments in the long-term interest rate, which in turn destabilize expectations. In this case, agents anticipate sharp future corrections and continuously revise their forecasting rules, preventing the system from converging.

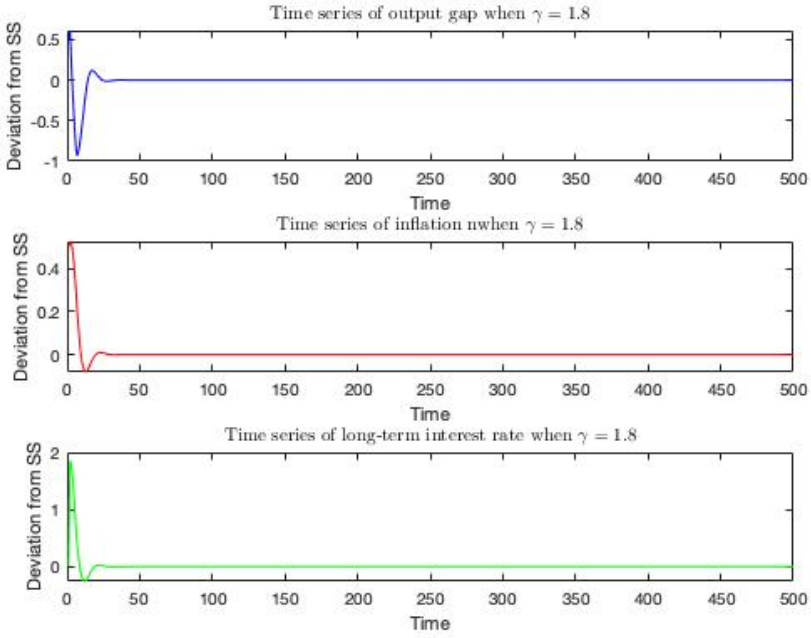


Figure 7: Time series of the output gap, inflation, and long-term interest rate under monetary dominance ($\phi_\pi = 1.5$) and passive fiscal policy ($\zeta = 0.4$).

A second configuration under monetary dominance that delivers stability arises when the portfolio friction parameter between short- and long-term assets is increased. In fact, higher frictions reduce the degree of arbitrage between short and long rates, thereby mitigating the transmission of shocks across maturities. As a result, the feedback loop from debt to long-term rates becomes less pronounced, expectations adjust more smoothly, and the system is able to converge. In this case, the friction parameter acts as a sort of anchoring mechanism: by limiting reallocations between maturities, it reduces fluctuations in long-term rates and contributes to overall stability, as shown in Figure 8.

In summary, under monetary dominance the system displays instability, but this instability can be mitigated either when fiscal policy responds in an intermediate and credible manner to the debt stock or when portfolio arbitrage across maturities is limited, both mechanisms allowing expectations to stabilize and supporting convergence to the equilibrium.

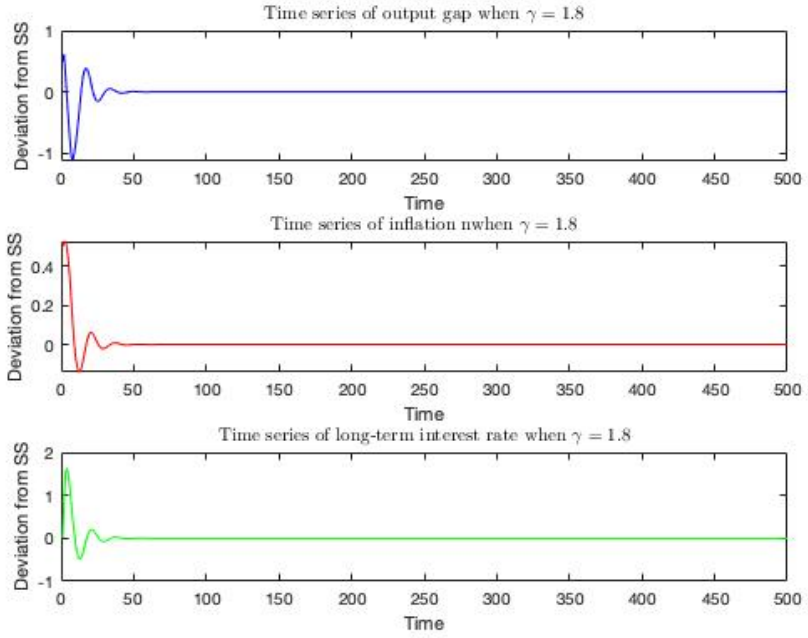


Figure 8: Time series of the output gap, inflation, and long-term interest rate under monetary dominance ($\phi_\pi = 1.5$) and passive fiscal policy ($\zeta = 0.01$) and $\Phi = 0.2$.

7 Long-term Interest Rate and Fractions Dynamics

In order to better investigate the complete dynamics, it is useful to look at the time series of our variable of interest-the long-term interest rate-and of the fraction of agents using the fundamental, the optimistic and the pessimistic rules. We start to investigate what happens to the dynamic behavior of the system by considering a moderate passive fiscal policy ($\zeta = 0.25$) when the intensity of choice is low ($\gamma = 0$). In this case, agents in the economy have no propensity to switch between different forecasting strategies and they are extremely conservative in their decision-making. As a result, the long-term interest rate converges to the steady state (as shown in Figure 9). Therefore, when examining the system of Equations (53), it is clear that the first three terms on the right-hand side disappear. As a result, the third equation in the system reduces to only the fourth and fifth terms, indicating that the long-term interest rate is determined solely by the past period's short-term interest rate and the short-term government debt.

When the intensity of choice is increased to an intermediate value ($\gamma = 1.8$), the dynamic

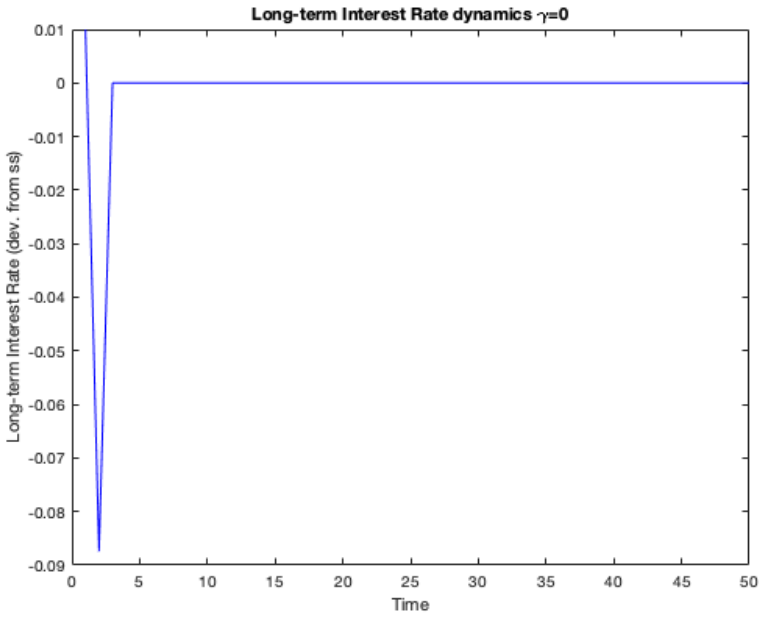


Figure 9: Long-term interest rate converge to the steady state level when $\gamma = 0$ ($\zeta = 0.25$).

behavior changes, as evident in Figure 10.

In this latter case, agents are able to identify which forecasting strategy performs better and react more aggressively by switching toward the most successful predictor. After an initial transient phase, during which the dynamics depend upon the choice of initial conditions, the system converges to a two-cycle, characterized by alternating periods of high and low interest rates. In this evolutionary environment with a higher intensity of choice, the three types of agents coexist, with time-varying fractions, and the long-term interest rate fluctuates around the unstable fundamental steady state. Along the cycle, the fraction of fundamentalists remains relatively small, while the market is dominated by optimists and pessimists depending on the phase of the cycle.

The rapid switching behavior between these two groups generates pronounced volatility in the long-term interest rate, leading to large oscillations around the steady-state value. In this setting, agents systematically commit forecasting errors, predicting high (low) interest rates in periods in which the realized interest rate turns out to be low (high).

We now consider the dynamics of the system under a more responsive fiscal policy, with $\zeta = 0.75$, as illustrated in Figure 11. The figure shows that, when the fiscal rule responds more

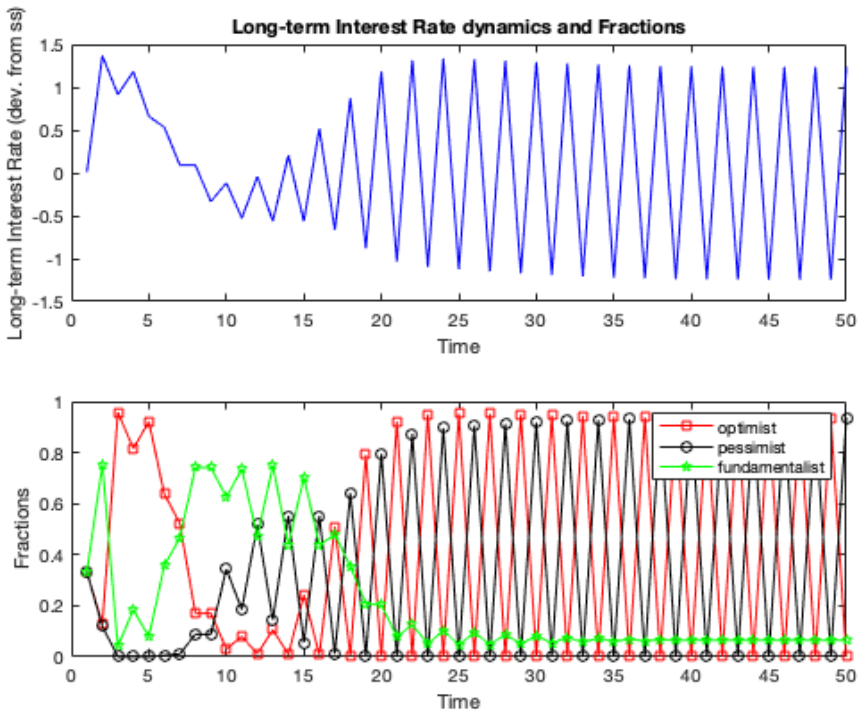


Figure 10: Long-term Interest Rate dynamics and fractions when $\gamma = 1.8$ ($\zeta = 0.25$).

aggressively to public debt, the resulting dynamics become more complex to analyze.

In this scenario, the fraction of fundamentalists fluctuates with a larger amplitude than in the previous case, but it is still not sufficient to drive alternative forecasting rules out of the market. The model generates endogenous cycles driven by the switching mechanism across heuristics. The resulting dynamics follow a slow up-and-down process that never converges to the steady state. During some periods, expectations are reinforced, while in others they are weakened. The system again exhibits relatively rapid dynamics, inducing agents to frequently revise their expectations in response to swift changes in economic conditions. However, compared with the previous case, the speed of agents' switching behavior is slightly reduced, and oscillations around the steady state are at times less pronounced. When the fiscal rule is less responsive to public debt, the coefficient on previously issued short-term debt (b_{t-1}) in the long-term interest rate equation of system (53) becomes negative, substantially affecting the dynamics of the interest rate. In this case, the negative coefficient on previously issued debt implies that the debt-related component of the long-term interest rate exerts a downward pressure on R_t^L. This effect dominates the positive

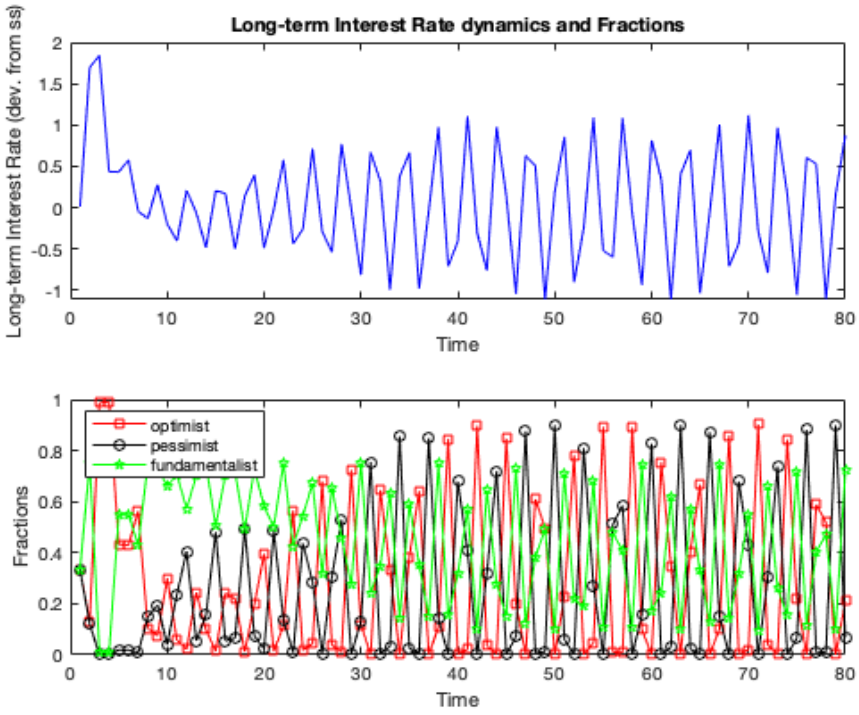


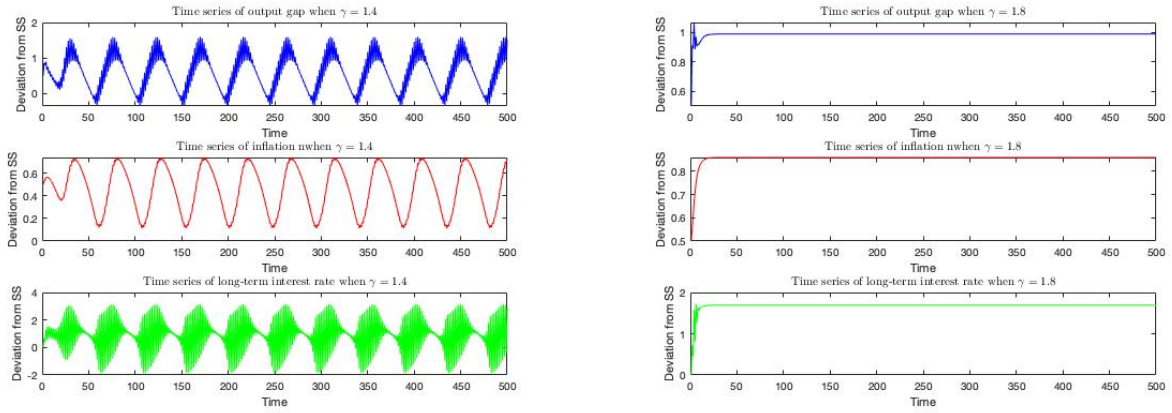
Figure 11: Long-term Interest Rate dynamics and fractions when $\gamma = 1.8$ ($\zeta = 0.75$)

contributions arising from expectations of the output gap and inflation, generating an immediate reversal in the trajectory of the long-term interest rate. This mechanism is less pronounced under a more responsive fiscal rule, for which the corresponding parameter becomes positive. In this case, the system continues to display oscillatory behavior; however, the dynamics are characterized by less pronounced fluctuations and slower trend reversals.

8 Equilibrium Dynamics under Fiscal Dominance

We now focus on the fiscal-dominance configuration and investigate how the system behaves when fiscal policy is active and monetary policy adjusts passively. This allows us to assess whether the mechanisms driving instability under monetary dominance persist or differ once the policy roles are reversed. Under fiscal dominance regime, the system displays chaotic instability for low levels of agents' sensitivity, similarly to what occurs under monetary dominance. However, as sensitivity of agents increases, hence the switching behavior is more pronounced,

the system converges to an alternative equilibrium, implying that stronger behavioral reactions alter the long-run adjustment path and the dominant forecasting rule is anchored away from the targeted steady state, as shown in Figure 12.



(a) **(b)**

Figure 12: Panel (a) shows the time series of the output gap, inflation, and long-term interest rate for $\gamma = 1.4$. Panel (b) considers a higher level of agents' sensitivity, $\gamma = 1.8$.

Finally, even under fiscal dominance, chaotic instability does not necessarily prevail. When the portfolio friction between short- and long-term assets increases (in particular for values of $\Phi > 0.18$ as before), the system is again able to converge to the fundamental equilibrium. The mechanism is similar to the one identified under monetary dominance: higher frictions limit arbitrage, thereby reducing the strength of the feedback from debt to long-term interest rates. As this feedback weakens, expectations become better anchored and switching across forecasting rules is reduced, leading to smoother dynamics and allowing the economy to remain closer to the targeted steady state. In this sense, higher portfolio frictions operate as a stabilizing mechanism by preventing excessive adjustments in long-term interest rates and smoothing the dynamics of the system, as illustrated in Figure 13.

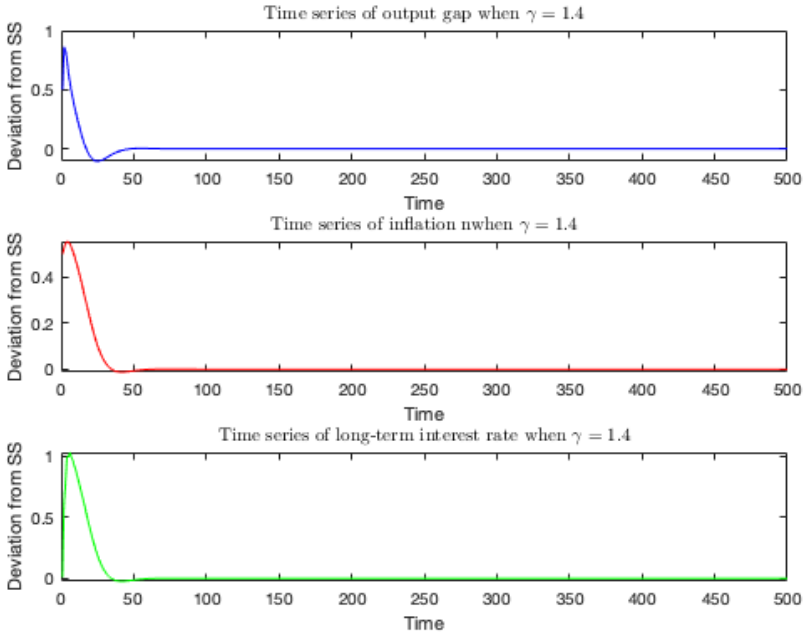


Figure 13: Time series of the output gap, inflation, and long-term interest rate under fiscal dominance and $\Phi = 0.2$.

9 Concluding Remarks

The issue of interaction between monetary and fiscal policies is a central topic in macroeconomic theory, but is largely unexplored when agents are allowed to hold heterogeneous and boundedly rational expectations. The existing literature has traditionally emphasized the stabilizing properties of alternative policy regimes—especially in the context of monetary dominance versus fiscal dominance—under the assumption of forward-looking agents with rational expectations. Within that framework, well-designed feedback rules are sufficient to ensure equilibrium uniqueness and local determinacy. This paper has shown that these conclusions do not necessarily extend to environments in which expectations are heterogeneous and evolve endogenously over time.

By embedding an evolutionary selection of forecasting rules into a New Keynesian model with a term-structure dimension and financial frictions, we have demonstrated that neither adherence to the Taylor principle nor the implementation of active fiscal policy consistent with the fiscal theory of the price level guarantees macroeconomic stability. Under both monetary and

fiscal dominance, heterogeneous expectations may generate endogenous instability in the form of persistent fluctuations or attracting cycles, even when policy rules would ensure determinacy under rational expectations. From a policy perspective, these results highlight that expectation heterogeneity is not a mere technical refinement, but a key determinant of aggregate dynamics.

A central insight of the paper is that equilibrium indeterminacy in this framework is intimately related to the evolutionary dynamics governing agents' beliefs. When agents switch sufficiently aggressively between forecasting strategies on the basis of relative past performance, the economy becomes vulnerable to expectation-driven fluctuations. In this sense, policy-induced instability emerges not from exogenous shocks, but from the endogenous interaction between policy rules and adaptive belief formation. This finding underscores the importance of accounting explicitly for expectation coordination when evaluating the performance of monetary and fiscal regimes.

The analysis also delivers clear implications for policy design. While a standard Ricardian fiscal policy is sufficient to rule out explosive debt paths, it is not sufficient to ensure equilibrium uniqueness in an economy populated by heterogeneous forecasters. Our results indicate that a regime of monetary dominance combined with a strong fiscal reaction of primary surpluses to debt accumulation is particularly effective in stabilizing the economy. Such a policy configuration dampens the scope for self-fulfilling belief dynamics by anchoring expectations and coordinating the evolutionary selection of forecasting rules, thereby reducing the likelihood of cyclical equilibria.

Finally, the paper highlights the role of financial frictions as a stabilizing force. Stronger portfolio adjustment costs, by limiting arbitrage opportunities across maturities, weaken the feedback loop between expectations and macroeconomic outcomes. As a result, financial frictions tend to enlarge the determinacy region under both monetary and fiscal dominance and mitigate expectation-driven fluctuations. This finding suggests that financial market imperfections, often viewed as a source of instability, may instead contribute to macroeconomic stabilization in the presence of heterogeneous expectations. From a policy perspective, our results provide support

to the view that that appropriately designed monetary–fiscal interactions, together with a regulatory environment that limits excessive arbitrage in bond markets, can help contain expectation-driven fluctuations and enhance macroeconomic stability.

Taking stock, this paper contributes to the growing literature that challenges the robustness of policy prescriptions derived under rational expectations. Our findings suggest that it is essential for the design of monetary and fiscal rules to explicitly account for how heterogeneous expectations are formed and coordinated in the economy. Future research could extend the analysis by considering alternative learning mechanisms, richer forms of expectation heterogeneity, or the interaction between heterogeneous expectations and large, nonlinear shocks. Such extensions would further enhance our understanding of macroeconomic stability in environments that more closely resemble real-world decision making.

Declaration of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

A. Agliari, N. Pecora, and A. Spelta. Coexistence of equilibria in a new keynesian model with heterogeneous beliefs. *Chaos, Solitons & Fractals*, 79:83–95, 2015.

A. Agliari, F. Bignami, and N. Pecora. Nonlinear dynamics of a simple behavioral model of inflation. *Decisions in Economics and Finance*, pages 1–25, 2025.

J. Andrés, J. D. López-Salido, and E. Nelson. Tobin's imperfect asset substitution in optimizing general equilibrium. *Journal of Money, Credit and Banking*, pages 665–690, 2004.

M. Anufriev and C. Hommes. Evolutionary selection of individual expectations and aggregate outcomes in asset pricing experiments. *American Economic Journal: Microeconomics*, 4(4):35–64, 2012.

M. Anufriev, T. Assenza, C. Hommes, and D. Massaro. Interest rate rules and macroeconomic stability under heterogeneous expectations. *Macroeconomic Dynamics*, 17(8):1574–1604, 2013.

M. Anufriev, F. Lamantia, D. Radi, and T. Tichy. Leaning against the wind in the new keynesian model with heterogeneous expectations. *Journal of Economic Dynamics and Control*, 172: 104993, 2025.

G. Ascari, N. Pecora, and A. Spelta. Boom and burst in housing market with heterogeneous agents. Technical report, Quaderni di Dipartimento, 2012.

T. Assenza and M. Berardi. Learning in a credit economy. *Journal of Economic Dynamics and Control*, 33(5):1159–1169, 2009.

T. Assenza, P. Heemeijer, C. H. Hommes, and D. Massaro. Individual expectations and aggregate macro behavior. *Tinbergen Institute discussion paper 13-016/II*, 2013.

T. Assenza, P. Heemeijer, C. Hommes, and D. Massaro. Managing self-organization of expectations through monetary policy: A macro experiment. *Journal of Monetary Economics*, 117: 170–186, 2021.

F. Bianchi, R. Faccini, and L. Melosi. A fiscal theory of persistent inflation. *The Quarterly Journal of Economics*, 138(4):2127–2179, 2023.

H. Bohn. The sustainability of budget deficits in a stochastic economy. *Journal of Money, credit and banking*, 27(1):257–271, 1995.

H. Bohn. The sustainability of fiscal policy in the united states. *Sustainability of public debt*, 15: 15–50, 2008.

M. D. Bordo, J. H. Cochrane, and J. B. Taylor. *Getting monetary policy Back on track*. Hoover Institution Press, 2024.

W. A. Branch. The theory of rationally heterogeneous expectations: evidence from survey data on inflation expectations. *The Economic Journal*, 114(497):592–621, 2004.

W. A. Branch. Sticky information and model uncertainty in survey data on inflation expectations. *Journal of Economic Dynamics and Control*, 31(1):245–276, 2007.

W. A. Branch and G. W. Evans. Intrinsic heterogeneity in expectation formation. *Journal of Economic theory*, 127(1):264–295, 2006.

W. A. Branch and B. McGough. A new keynesian model with heterogeneous expectations. *Journal of Economic Dynamics and Control*, 33(5):1036–1051, 2009.

W. A. Branch and B. McGough. Dynamic predictor selection in a new keynesian model with heterogeneous expectations. *Journal of Economic Dynamics and Control*, 34(8):1492–1508, 2010.

A. Brazier, R. Harrison, M. King, and T. Yates. The danger of inflating expectations of macroeconomic stability: Heuristic switching in an overlapping-generations monetary model. *Thirteenth issue (June 2008) of the International Journal of Central Banking*, 2018.

W. A. Brock and C. H. Hommes. A rational route to randomness. *Econometrica: Journal of the Econometric Society*, pages 1059–1095, 1997.

J. Bullard and K. Mitra. Learning about monetary policy rules. *Journal of monetary economics*, 49 (6):1105–1129, 2002.

J. Bullard, A. Grimaud, I. Salle, and G. Vermandel. Soft landing and inflation scares. *Journal of Monetary Economics*, page 103871, 2025.

G. A. Calvo. Staggered prices in a utility-maximizing framework. *Journal of monetary Economics*, 12(3):383–398, 1983.

M. Canzoneri, R. Cumby, and B. Diba. The interaction between monetary and fiscal policy. *Handbook of monetary economics*, 3:935–999, 2010.

C. D. Carroll. Macroeconomic expectations of households and professional forecasters. *the Quarterly Journal of economics*, 118(1):269–298, 2003.

J. Cochrane. *The fiscal theory of the price level*. Princeton University Press, 2023.

J. H. Cochrane. A frictionless view of us inflation. *NBER macroeconomics annual*, 13:323–384, 1998.

J. H. Cochrane. Money as stock. *Journal of Monetary Economics*, 52(3):501–528, 2005.

P. De Grauwe. Animal spirits and monetary policy. *Economic theory*, 47:423–457, 2011.

P. De Grauwe and M. Grimaldi. Exchange rate puzzles: a tale of switching attractors. *European Economic Review*, 50(1):1–33, 2006.

A. K. Dixit and J. E. Stiglitz. Monopolistic competition and optimum product diversity. *The American economic review*, 67(3):297–308, 1977.

G. W. Evans and S. Honkapohja. Adaptive learning and monetary policy design. *Journal of Money, Credit and Banking*, pages 1045–1072, 2003a.

G. W. Evans and S. Honkapohja. Expectations and the stability problem for optimal monetary policies. *The Review of Economic Studies*, 70(4):807–824, 2003b.

G. W. Evans and S. Honkapohja. Monetary policy, expectations and commitment. *Scandinavian Journal of Economics*, 108(1):15–38, 2006.

G. W. Evans, S. Honkapohja, and R. Marimon. Convergence in monetary inflation models with heterogeneous learning rules. *Macroeconomic Dynamics*, 5(1):1–31, 2001.

M. Falagiarda. Evaluating quantitative easing: a dsge approach. *International Journal of Monetary Economics and Finance*, 7(4):302–327, 2014.

J. Galí. *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*. Princeton University Press, 2015.

J. Galì. Rethinking the new keynesian model. *University of Cambridge*, 2025.

M. Guerini. Is the friedman rule stabilizing? some unpleasant results in a heterogeneous expectations framework. Technical report, Working Paper, 2013.

R. Harrison. Asset purchase policy at the effective lower bound for interest rates. *Bank of England Working Paper No. 678*, 2012.

R. Harrison. Optimal quantitative easing. *Bank of England Working Paper No. 444*, 2017.

C. Hommes. The heterogeneous expectations hypothesis: Some evidence from the lab. *Journal of Economic dynamics and control*, 35(1):1–24, 2011.

C. Hommes and J. Lustenhouwer. Inflation targeting and liquidity traps under endogenous credibility. *Journal of Monetary Economics*, 107:48–62, 2019.

C. H. Hommes. Heterogeneous agent models in economics and finance. *Handbook of computational economics*, 2:1109–1186, 2006.

M. Kurz and M. Motolese. Diverse beliefs and time variability of risk premia. *Economic Theory*, 47(2-3):293–335, 2011.

M. Kurz, G. Piccillo, and H. Wu. Modeling diverse expectations in an aggregated new keynesian model. *Journal of Economic Dynamics and Control*, 37(8):1403–1433, 2013.

B. LeBaron. Agent-based computational finance. *Handbook of computational economics*, 2:1187–1233, 2006.

E. M. Leeper. Equilibria under 'active'and 'passive'monetary and fiscal policies. *Journal of monetary Economics*, 27(1):129–147, 1991.

E. M. Leeper and C. Leith. Understanding inflation as a joint monetary–fiscal phenomenon. In *Handbook of macroeconomics*, volume 2, pages 2305–2415. Elsevier, 2016.

L. Ljungqvist and T. J. Sargent. Recursive macroeconomic theory second edition, 2004.

T. Lux and M. Marchesi. Volatility clustering in financial markets: a microsimulation of interacting agents. *International journal of theoretical and applied finance*, 3(04):675–702, 2000.

N. G. Mankiw, R. Reis, and J. Wolfers. Disagreement about inflation expectations. *NBER macroeconomics annual*, 18:209–248, 2003.

C. F. Manski and D. McFadden. Alternative estimators and sample designs for discrete choice analysis. *Structural analysis of discrete data with econometric applications*, 2:2–50, 1981.

M. Marzo and P. Zagaglia. Macroeconomic stability in a model with bond transaction services. *International Journal of Financial Studies*, 6(1):23, 2018.

F. Matějka and A. McKay. Rational inattention to discrete choices: A new foundation for the multinomial logit model. *American Economic Review*, 105(1):272–298, 2015.

B. Preston. Adaptive learning, forecast-based instrument rules and monetary policy. *Journal of Monetary Economics*, 53(3):507–535, 2006.

C. A. Sims. A simple model for study of the determination of the price level and the interaction of monetary and fiscal policy. *Economic theory*, 4:381–399, 1994.

J. B. Taylor. Discretion versus policy rules in practice. In *Carnegie-Rochester conference series on public policy*, volume 39, pages 195–214. Elsevier, 1993.

J. B. Taylor. A historical analysis of monetary policy rules. In *Monetary policy rules*, pages 319–348. University of Chicago Press, 1999.

D. Vayanos and J.-L. Vila. A preferred-habitat model of the term structure of interest rates. *National Bureau of Economic Research*, 2009.

M. Woodford. Monetary policy and price level determinacy in a cash-in-advance economy. *Economic theory*, 4(3):345–380, 1994.

M. Woodford. Price-level determinacy without control of a monetary aggregate. In *Carnegie-Rochester conference series on public policy*, volume 43, pages 1–46. Elsevier, 1995.

M. Woodford. Fiscal requirements for price stability. *National Bureau of Economic Research Cambridge, Mass., USA*, 2001.

M. Woodford. *Interest and Prices*. Princeton University Press, 2003.

M. Woodford and C. E. Walsh. Interest and prices: Foundations of a theory of monetary policy. *Macroeconomic Dynamics*, 9(3):462–468, 2005.

A Appendix

A.1 Households

The maximization problem for the household is:

$$U_t = E_t \sum_{t=0}^{\infty} \beta^t u\Big(C_t, N_t\Big) \tag{54}$$

where u is defined as:

$$u\Big(C_t, N_t\Big) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi} \tag{55}$$

subject to

$$B_t^L + B_t = R_{t-1}B_{t-1} + R_t^{1,L}B_{t-1}^L + W_t N_t + D_t - T_t - P_t C_t - \frac{\Phi P_t(b+b^L)}{2}\left[\delta\frac{B_t}{B_t^L} - 1\right]^2 \tag{56}$$

Let the real bond holdings be defined as:

$$b_t = \frac{B_t}{P_t} \tag{57}$$

$$b_t^L = \frac{B_t^L}{P_t} \tag{58}$$

We can rewrite the problem in real terms and the budget constraint becomes:

$$b_t^L + b_t = R_{t-1}\frac{b_{t-1}}{\Pi_t} + \frac{b_{t-1}^L}{\Pi_t}R_t^{1,L} + \frac{W_t}{P_t}N_t - \frac{T_t}{P_t} + \frac{D_t}{P_t} - C_t - \frac{\Phi(b+b^L)}{2}\left[\delta\frac{b_t}{b_t^L} - 1\right]^2 \tag{59}$$

The first-order conditions (w.r.t. C_t, N_t, B_t and B_t^L) for the optimization problem are:

$$\lambda_t = C_t^{-\sigma} \tag{60}$$

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\psi} \tag{61}$$

$$-\lambda_t + \beta \frac{R_t}{\Pi_{t+1}} E_t \lambda_{t+1} - \lambda_t \Phi(b + b^L) \left[\delta \frac{b_t}{b_t^L} - 1 \right] \frac{\delta}{b_t^L} = 0 \tag{62}$$

$$-\lambda_t + \beta \frac{R_{t+1}^{1,L}}{\Pi_{t+1}} E_t \lambda_{t+1} - \lambda_t \Phi(b + b^L) \left[\delta \frac{b_t}{b_t^L} - 1 \right] \left[\frac{\delta b_t}{(b_t^L)^2} \right] = 0 \tag{63}$$

Some steady state relationship could be useful for the log-linearization of the previous expressions:

$$\frac{b^L}{b} = \delta \tag{64}$$

$$\frac{b}{b + b^L} = (1 + \delta)^{-1} \tag{65}$$

$$\frac{b^L}{b + b^L} = 1 - (1 + \delta)^{-1} = \frac{\delta}{1 + \delta} \tag{66}$$

$$\beta = R^{-1} \tag{67}$$

Combining Equations (60) and (62), we get a consumption Euler equation:

$$C_t^{-\sigma} = \beta \frac{R_t}{\Pi_{t+1}} C_{t+1}^{-\sigma} - C_t^{-\sigma} \Phi \delta (1 + \delta) \left[\delta \frac{b_t}{b_t^L} - 1 \right] \tag{68}$$

which can be log-linearized to give:

$$\hat{c}_t = \hat{c}_{t+1} - \frac{1}{\sigma} (\hat{R}_t - E_t \hat{\Pi}_{t+1}) + \frac{\delta (1 + \delta)}{\sigma} \Phi (\hat{b}_t - \hat{b}_t^L) \tag{69}$$

The FOCs for labor supply (61) and consumption (60) can be combined and log-linearized to give:

$$\hat{\omega}_t = \sigma \hat{c}_t + \psi \hat{n}_t \tag{70}$$

Log-linearizing the FOC w.r.t the long-term government bond (63) gives:

$$\hat{c}_t = \hat{c}_{t+1} - \frac{1}{\sigma}(E_t\hat{R}^L_{t+1} - E_t\hat{\Pi}_{t+1}) + \frac{(1+\delta)}{\sigma}\Phi(\hat{b}_t - \hat{b}^L_t) \tag{71}$$

Adding equations (71) and (69) gives:

$$E_t\hat{R}^L_{t+1} = \hat{R}_t - \Phi(1+\delta)^2(\hat{b}_t - \hat{b}^L_t) \tag{72}$$

The last equation can be used to write the Euler equation in terms of returns on short-term and long-term bonds. In particular, we rewrite Equation (72) as:

$$\hat{R}_t - E_t\hat{R}^L_{t+1} = \Phi(1+\delta)^2(\hat{b}_t - \hat{b}^L_t) \tag{73}$$

The LHS of this last expression appears on the RHS of Equation (69). The consumption Euler Equation can be written as:

$$\hat{c}_t = \hat{c}_{t+1} - \frac{1}{\sigma}\left(\frac{1}{1+\delta}\hat{R}_t - E_t\hat{\Pi}_{t+1} + \frac{\delta}{1+\delta}E_t\hat{R}^L_{t+1}\right) \tag{74}$$

A.2 Derivation of the State-Space Representation

In order to compute simulations with MATLAB, we need to rewrite the system of equations in the state-space form, meaning that today's variables have to be functions of past values and expectations. The system of the equations is:

$$
\begin{cases}
\hat{x}_t = E_t\hat{x}_{t+1} - \frac{1}{\sigma}\left(\frac{1}{1+\delta}\hat{R}_t - E_t\hat{\pi}_{t+1} - r_t^n + \frac{\delta}{1+\delta}E_t\hat{R}_{t+1}^L\right) \\
\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \kappa\hat{x}_t \\
\hat{R}_t^L = \frac{\beta}{\delta}\hat{b}_t + \beta b_t^L + [\beta\zeta(1+\delta) - 1]\delta^{-1}\hat{b}_{t-1} + \hat{R}_{t-1} - \hat{b}_{t-1}^L + \frac{1+\delta}{\delta}\hat{\pi}_t \\
\hat{b}_t = \frac{1}{\Phi(1+\delta)^2}\hat{R}_t - \frac{1}{\Phi(1+\delta)^2}\hat{R}_{t+1}^{1,L} + \hat{b}_t^L \\
\hat{R}_t = \phi_\pi\hat{\pi}_t + \phi_x\hat{x}_t + v_t
\end{cases} \tag{75}
$$

The system can be expressed in state-space form as follows. As before, log-linearized variables are hereafter denoted by lowercase letters.

$$
A\begin{pmatrix} x_t \\ \pi_t \\ R_t^{1,L} \\ b_t \\ R_t \end{pmatrix} = B\begin{pmatrix} E_t x_{t+1} \\ E_t\pi_{t+1} \\ E_t R_{t+1}^{1,L} \\ E_t b_{t+1} \\ E_t R_{t+1} \end{pmatrix} + C\begin{pmatrix} x_{t-1} \\ \pi_{t-1} \\ R_{t-1}^{1,L} \\ b_{t-1} \\ R_{t-1} \end{pmatrix} + M \tag{76}
$$

where $A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ -\kappa & 1 & 0 & 0 & 0 \\ 0 & -\frac{1+\delta}{\delta} & 1 & -\frac{\beta}{\delta} & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{\Phi(1+\delta)^2} \\ -\phi_x & -\phi_\pi & 0 & 0 & 1 \end{pmatrix}$,

$$
B = \begin{pmatrix} 1 & \frac{1}{\sigma} & -\frac{\delta}{\sigma(1+\delta)} & 0 & 0 \\ 0 & \beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{\Phi(1+\delta)^2\delta} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{[\beta(1+\delta)\zeta-1]}{\delta} & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, M = \begin{pmatrix} \frac{1}{\sigma}r_t^n \\ 0 \\ \beta b_{t-1}^L - b_t^L \\ b_t^L \\ v_t \end{pmatrix}.
$$

After computing A^{-1} and verifying that it is invertible, we premultiply both sides of the system by A^{-1}. The system to be analyzed can then be written in its final form. We focus on the matrices that govern the system's dynamic properties and on which the stability analysis is based:

$$\begin{pmatrix} x_t \\ \pi_t \\ R_t^{1,L} \\ b_t \\ R_t \end{pmatrix} = r \begin{pmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 \\ a_{41} & a_{42} & a_{43} & 0 & 0 \\ a_{51} & a_{52} & a_{53} & 0 & 0 \end{pmatrix} \begin{pmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \\ E_t R_{t+1}^{1,L} \\ E_t b_{t+1} \\ E_t R_{t+1} \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_{34} & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_{t-1} \\ \pi_{t-1} \\ R_{t-1}^{1,L} \\ b_{t-1} \\ R_{t-1} \end{pmatrix} \tag{77}$$

where:

$$r = \frac{1}{(1+\delta)\sigma + \phi_x + \kappa\phi_\pi},$$

$$a_{11} = \sigma(1+\delta),$$

$$a_{12} = (1+\delta) - \beta\phi_\pi,$$

$$a_{13} = -\delta,$$

$$a_{21} = \sigma\kappa(1+\delta),$$

$$a_{22} = \beta(\sigma + \phi_x + \delta\sigma) + \kappa(1+\delta),$$

$$a_{23} = -\delta\kappa,$$

$$a_{31} = \frac{\sigma\Phi\kappa\delta(\delta^2+3\delta+3) + \Phi\kappa\sigma + \beta\sigma(\phi_x + \kappa\phi_\pi)}{\delta\Phi(1+\delta)},$$

$$a_{32} = \frac{\Phi\kappa\delta(\delta^2+3\delta+3) + \Phi\kappa + \beta(\phi_x + \kappa\phi_\pi) + \beta\Phi\sigma(\delta^3+3\delta^2+3\delta+1) + \beta\Phi\phi_x(1+\delta)^2 + \beta\phi_\pi\sigma}{\delta\Phi(1+\delta)},$$

$$a_{33} = \frac{\sigma\kappa + \beta(\phi_x + \kappa\phi_\pi) - \delta^2\Phi\kappa(\delta^2+3\delta+3) - \beta\left[\sigma(1+\delta) + \phi_x + \kappa\phi_\pi\right]}{\delta\Phi(1+\delta)^2},$$

$$a_{41} = \frac{\sigma(\phi_x + \kappa\phi_\pi)}{\Phi(1+\delta)},$$

$$a_{42} = \frac{\phi_x + \kappa\phi_\pi + \beta\phi_\pi\sigma}{\Phi(1+\delta)},$$

$$a_{43} = \frac{\delta(\phi_x + \kappa\phi_\pi) + \left[\sigma(1+\delta) + \phi_x + \kappa\phi_\pi\right]}{\Phi(1+\delta)^2\left[\sigma(1+\delta) + \phi_x + \kappa\phi_\pi\right]},$$

$$a_{51} = \sigma(\phi_x + \kappa\phi_\pi)(1+\delta),$$

$$a_{52} = \beta(\phi_\pi \sigma + \delta \phi_\pi \sigma) + (\phi_x + \kappa \phi_\pi)(1 + \delta),$$

$$a_{53} = -\delta(\phi_x + \kappa \phi_\pi),$$

$$b_{34} = \big[\beta\zeta(1 + \delta) - 1\big]\delta^{-1}.$$

A.3 Simulations for high levels of γ

In this situation, in every period, all the agents immediately switch towards the predictor that in the previous period had shown the lowest forecast error. If we look at the phase plot for output gap, inflation and long-term interest rate we see that they may converge two different steady states depending on whether the initial conditions are nearby or far from the fundamental steady state. In particular, the graph at the top shows that the system converges to the fundamental steady state for a set of initial conditions $(x_0, \pi_0, R_0^L) \in [-0.4; 0.4]$;the second graph (at the bottom) shows that convergence to a different stable steady state occurs for initial conditions that differ from the previous set.

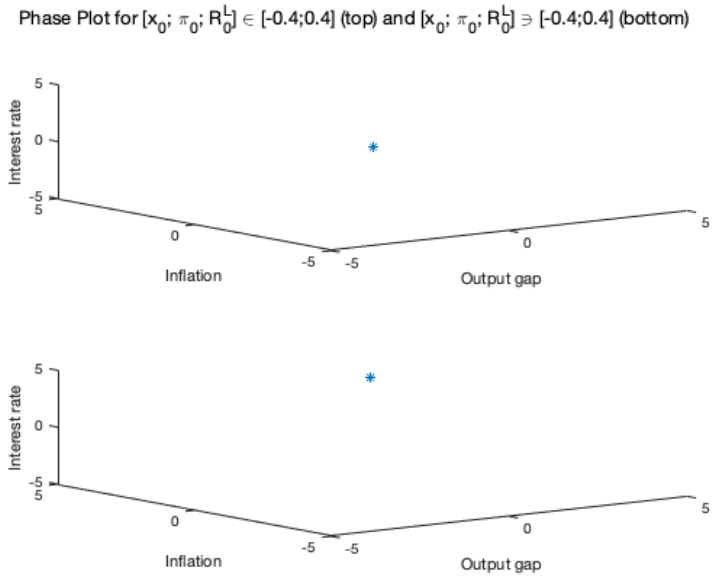


Figure 14: Phase Plot for $\gamma = 50$